THE USE OF
THE RULE OF
PROPORTION:
In *Arithmetique* and *Geometrie*:

Firſt publiſhed at Paris *in the* French *tongue, and dedicated to* Monſieur, *the then Kings only brother,* (*now Duke of Orleans,*

By *Edm: wingate an Engliſh Gent.*

And now tranſlated into *Engliſh* by the ſame Author:

Whereinto is now alſo inſerted the Conſtruction *of the ſame Rule, and a farther uſe thereof, in queſtions that concern*

Aſtronomie, Dialling, Geographie, Navigation, Gaging of Veſſell, Military Orders, Intereſt and Annuities.

Eccleſiaſticus 39. 17.
None may ſay, What is this? wherefore is that? for at time convenient they ſhall all be ſought out.

LONDON,
Printed by *M. F.* for *P. Stephens* at the *gilded Lion in* Pauls *Churchyard.*

A TRES-HAUT ET Tres-puissant Prince, Monsieur, Gaston de France, Frere unique du Roy, Duc d'Anjou, &c.

Monseigneur,

QVelque temps apres mon arrivee en ceste ville, ayant fait voir l'Instrument, dont i'explique les utilitez en se liuret, & discouru de quelques uns de ses usages, i'appris de plusieurs, que si l'on travailloit sur ce suiet, que le labour en serroit bien recueilly: Cela (pour vous dire la veritè) m'a enhardy d'en dire quelque chose, & (luy faisant voir le iour) me targuer de vostre authoritè;

Vous me pardonerez toutesfois, ſi i'ay eu ceſte hardieſſe de luy donner du credit, & de la recommendation de ce coſtè la, comme i'ay eu la voluntè de teſmoigner combien ie ſuis,

Monſeigneur,

Voſtre treſ-humble
& treſ-obeyſſant
ſerviteur

EDM: WINGATE.

To

To the High and Mighty Prince *Monſieur*, Gaſton of France, the Kings onely Brother, Duke of Anjou, &c.

My Lord,

NOt long after my arrival in this City, having divulged the Inſtrument (whoſe uſes I explain in this little Treatiſe) and diſcourſed of ſome of the conveniences thereof, I was given to underſtand by divers, that if pains were beſtowed upon that ſubject, the labour therein taken might obtain good reception: This (to ſay truth) hath given me encouragement thereof to ſay

ſomewhat, and (having cauſed it to ſee the light) to ſhelter it under your protection: Neverthelesſe you ſhall pardon me, for that by preſuming to procure unto it from thence credit & recommendation I have expreſſed a willingneſſe to teſtifie, how much I am,

My Lord,

Your moſt humble and

moſt obedient

Servant,

EDM: WINGATE.

An

An extract of the Priviledge of *Lewes* the XIII. Late King of France.

B*Y the Kings favour and priviledge*, Edmund Wingate (*an English Gentleman*) *is permitted to print, or cause to be printed, to sell, and distribute, certaine Tables called*, The Logarithmeticall Tables, the Construction & use of the same; *Also*, The construction, description, and use of the Rule of Proportion, which is framed by means of the said Tables; *And restraint is made to all Printers, Book-sellers, and others, to print, or cause to be printed the said Tables, &c. without the good will*

will and consent of the said Edm: Wingate, *or of those unto whom he shall grant his right, upon pain to forfeit unto us five hundred pounds* Parisis, *and the Copies which shall be found; as is more fully contained and declared in the Letters of the said Priviledge. Given at* Paris *the* 4. *day of* November, 1624. *And of our reign the* 15.

By the King in his Counsel,

Signed *HARDY.*

THE PREFACE TO THIS *Translation*.

AMongst the many rare effects produced by the noble invention of *Logarithmes*, the projection of the *Rule* of *Proportion* is not the least, which being first discovered by that Learned and Industrious Artist *Edm: Gunter* (late Professor of *Astronomy* in Gresham Colledge, London, deceased) was by me (in *Anno* 1624.) transported into *France*, and there communicated to most of the chiefest Mathematicians then residing in *Paris*, who apprehending the great benefit that might accrew thereby, importuned me to expresse the use thereof in the *French* tongue; which being performed accordingly, I was advised by Mr. *Alleaume* (the Kings chiefe Ingenier)

to dedicate my book to *Monsieur*, the then Kings onely brother, now Duke of *Orleans*: Neverthelesse this Work (as it was there published) comming forth as an *Abortive*, (the publishing thereof being somewhat hastned, by reason an Advocate of *Dijon* in Burgundy began to print some uses thereof, which I had in a friendly way communicated unto him) I thought it not worthy to see the light here in *England*, especially in regard Mr *Gunter* himself had learnedly explained the use thereof in a far larger Volume: Howbeit having now of late (by reason of the present troubles) had too much leisure from my other employments and calling, to look back to those studies, wherewith in my yonger time I used to busie my self; And having also upon that occasion bethought my self, how divers necessary additions might be fitly inserted into that Work, and many inconveniences in the use of that *Instrument*, which before did usually incumber the Practitioner, might be removed: I have adventured to let this Translation appear; In which you shall find expressed, as succinctly and plainly as I could, the use of that

Rule

Rule in the form as you find it annexed to this Book; Not that I would confine any man to uſe ſuch a form and no other, but becauſe the operations are thereupon underſtood and performed more perſpicuouſly and plainly, then (as I conceive) they would be, if the lines were thereupon otherwiſe deſcribed; Howbeit the uſe thereof being in this form once gained, the Practitioner may then uſe that way of deſcribing it, which ſorts beſt with his own humor.

Having thus acquainted you with the occaſion of publiſhing this Treatiſe, leſt I may now expoſe it to prejudice, give me leave to premiſe theſe few advertiſements following: Firſt, therefore, it is deſired, that he, who intends to reade this book with profit, ſhould have a proper *Genius* and Phanſie for the *Mathematiques*, not only ready to conceive *Mathematicall* notions; but likewiſe able to wreſtle with them, and apt to take pleaſure in them: For, *De quolibet ligno non fit Mercurius.* Again, it is expected he ſhould be aforehand furniſhed with competent knowledge in thoſe Sciences, *viz.* 1. In *Arithmetique* he ought to be acquainted with the

the nature of numbers, whole and broken, absolute and relative; with Numeration, Addition, Subtraction, Multiplication, Division, the Rule of three, direct & inverse; with the nature & extraction of Roots, square and cube; And with the right use of *Logarithmes*: 2. In *Geometry*, to be verst in the doctrine of Triangles, plain and sphericall, and (in some competent measure) to know their nature, together with the way and reason of their dimension; As also the dimension of other Geometricall figures: 3. In *Astronomy*, *Dialling*, and *Geography*, to understand that the Problemes which concern them, are resolved by the particular application of the doctrine of sphericall Triangles to those severall sciences: 4. In *Navigation*, to be indifferently wel read in such Authors as have explained that Art, and to be able therein also to make use of the doctrine of Triangles: With the knowledge of these things (I say) and the like he ought to be (in some reasonable sort) supplied, that intends to make a right and compleat use of this Treatise: For, none (I presume) will expect to find an intire body of the *Mathematiques* in this small

bulk,

bulk, which is onely intended for an *En-*
chiridion or Manuell of ſuch Mathemati-
call Rules and *Analogies*, as may moſt
properly ſerve for the reſolution of Pro-
blemes, which may be wrought upon
this *Inſtrument*: And therefore I wholly
eferre the *Reader* for demonſtrations
nd larger explanations of the matters in
this book contained, to the further ſcru-
iny of other Authors; Not doubting but
that (upon due peruſall hereof) he will
ind as much inſerted, as ſhall be thought
eceſſary to diſcover the manifold and
xquiſite uſe of the ſame *Inſtrument*. But
ere I would not be miſtaken, as if I did
otally exclude all others, who are not
repared with ſuch an univerſall know-
edge in the *Mathematiques*, from ha-
ing any capacity at all of underſtanding
his book; For, if he be only in part ac-
uainted with ſome of the abovementio-
ed Learning, he may be able to make uſe
f this *Inſtrument* according to that de-
ree of knowledge which he hath there-
n; For example, if he onely know *Mul-*
iplication and *Diviſion*, this Treatiſe
vill inſtruct him how to *multiply* and *di-*
vide upon the *Rule*, and ſo in like ſort of
the

the reſt: Howbeit (as I ſaid before) if he intend to have an intire underſtanding of the uſes of this *Inſtrument*, he muſt be alſo furniſhed with an intire knowledge of all the *Mathematiques*; becauſe it is ſubſervient to every branch of thoſe ſciences: And then the conveniency thereof will have ſuch latitude, that it will not be confined to thoſe uſes only promiſed in the Title of this Book, but likewiſe (by the variety of Rules and Examples therein found) may be readily and fitly applied to other Arts and Profeſſions not there remembred; As namely, in *Fortification*, the Ingenier may here be taught how to find the ſides of his *Polygonical* figures, the *Lines* of fortification according to the Rules of that Art, the *quantity* of Trenches and Ramparts, how to order and eſtimate the labour and work of Pioners, and the like: The *Surveyor* alſo may here furniſh himſelf with divers expeditious diſpatches, for the taking of diſtances, the ſumming up of Plots, being firſt divided into triangles, the diſtribution of Fields or Lordſhips to ſeverall perſons, the cutting off any part of a triangle or plot according to any quantity propounde

pounded, &c. The like may be ſaid of *Muſique, Architecture*, the *Perſpectives, Gunnery*, &c. The *Goldſmith* alſo, and *Mint-Maſter* may here learn how to tẽper their Allegations: The *Merchant* and *Tradeſman*, how to reſolve queſtions of Partnerſhip, and to caſt up the value of their commodities: The *Juſtice* of *Peace* and *High Conſtable*, how to rate a Town, Hundred, or County, &c. All which and much more muſt be wholly left to the diſcretion of thoſe, that will take the pains to underſtand the uſe of the ſaid *Inſtrument*; which (I perſwade my ſelf) no man (affecting the *Mathematiques*) will think much to undergoe, conſidering the benefit he may reap therby, and the delight he may take therein; For, by help thereof, and of a pair of Compaſſes, onely ſix Inches long, he may reſolve with requiſite exactneſſe any propoſition in the Arts and Sciences aboveremembred (which comes within the bounds of ordinary practiſe) without the help of pen or paper, and ſhall thereby alſo perform more in one howre, then otherwiſe (I mean by ordinary *Arithmetique*) he ſhall be able to diſpatch in two whole days.

But

But it may be objected, if this *Instrument* be of such excellent use as is here pretended, why hath it not been heretofore of greater esteem, it being now above twenty years since it was first invented? This objection may be answered divers ways: 1. It is no easie matter to drive men out of their old track, especially when they have entertained an opinion that there can be none better. 2. Again, the use thereof in the point of *Numbring* upon the *Rule* (which ought to be accounted the chiefest, and indeed the ground of all the rest) hath not been heretofore (under favour) so fully explained, as here you shall find it: For, albeit (I confesse) it were great presumption in me to assume to my self the reputation of having better abilities to describe any of the uses thereof, then M^r^ *Gunter* himself had, who first invented it; yet this I can averre upon mine own knowledge, that he did forbear to explain that use therof, because he took it for granted none would meddle with it but such only as were already well able to understand how to number upon it, having beforehand acquainted themselves with the

manner

manner of *numbring* upon *ſcales*, and with the nature of *Logarithmes:* For, when after my return out of *France*, I importuned him to make a fuller explanation, how to number upon it, to the end the uſe thereof might by that means be made more publique, his anſwer was, *that it could not be expected the Rule ſhould ſpeak*; Intimating thereby, that the Practitioner ſhould (in that point) rely much upon diſcretion, and not altogether depend upon precepts & examples. But laſtly the chiefeſt cauſes why this *Inſtrument* hath been hitherto obſcured and the uſes thereof no better known to the world, are theſe. 1. The Difficulty of deſcribing the lines thereupon with convenient exactneſſe: 2. The trouble of working thereupon by reaſon (ſometimes) of too large an extent of the Compaſſes: 3. The importableneſſe therof, it being requiſite for working upon ſuch a *Rule* (onely two foot long) to uſe a paire of Compaſſes of nine Inches: 4. The charge of purchaſing ſuch an *Inſtrument* made of braſſe or wood; For, none but ſuch have been heretofore uſed. For remedy of the firſt of theſe, I have

caused

caused the plate, whereupon this *Instrument* is printed, to be protracted with a great deal of care and circumspection, so that I dare affirme it to be as exactly drawn (for the main and most considerable divisions thereof) as may be expected from Art: For the second, having there three severall Lines of *Numbers* by degrees one lesse then another, when the Compasses are too little for one, you may use another, also *crosse-work* upon the Greatest line will prevent the too great extension of the Compasses; so that it will be requisite to use with this *Instrument* (as it is now contrived) a pair of Compasses only six Inches long, as I said before; & yet the divisions of this (I mean upon the great Line of *Numbers*) are neer as large againe, as those upon M^r^ *Gunters* Rule of the like length: The third and fourth impediments may also be remedied, if in stead of brasse or wood you use the impression of the said Plate upon Vellum or Imperiall paper, which may either be rolled up and couched in a little box, or otherwise pasted upon a Ruler, either flat, to use at home, or round, to be carried in a hollow staffe or Cane

together with the Compaſſes, which are to be uſed therewith. Alſo divers uſefull conveniences ſhall you meet withall in this Edition of the *Rule*; as namely, a readier way of finding out *Mean-Proportions*, the *Extraction* of Roots by Inſpection only, without aid of pen or compaſſes, and the like: For further diſcovery of all which, I refer you to the book it ſelf, hoping that my reall intention to advance the publique good will procure from the Ingenuous *Reader* a favourable conſtruction of what he ſhall therein find not wilfully miſtaken.

Grays Inne
Jan. 20,
164$\frac{4}{5}$.

The Contents.

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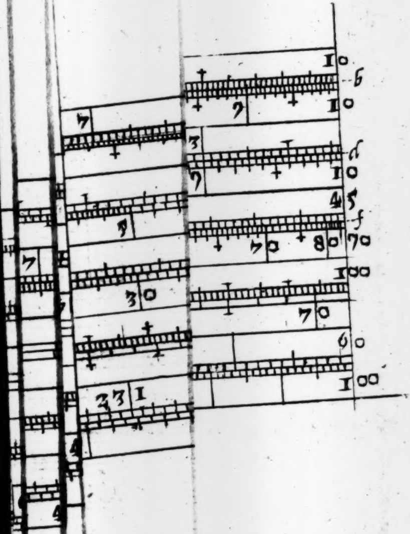

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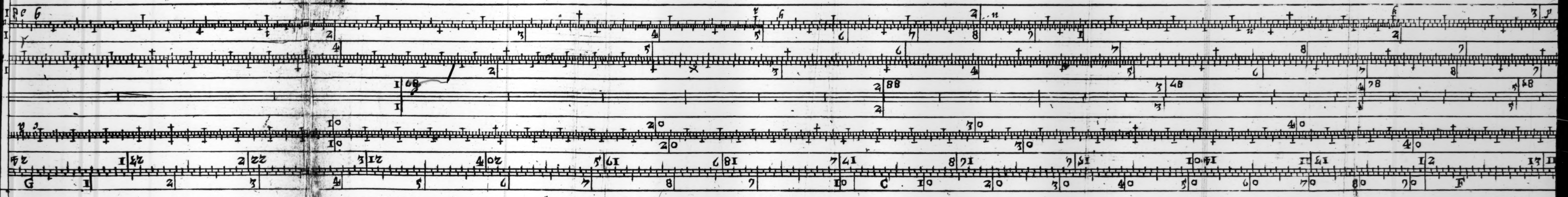
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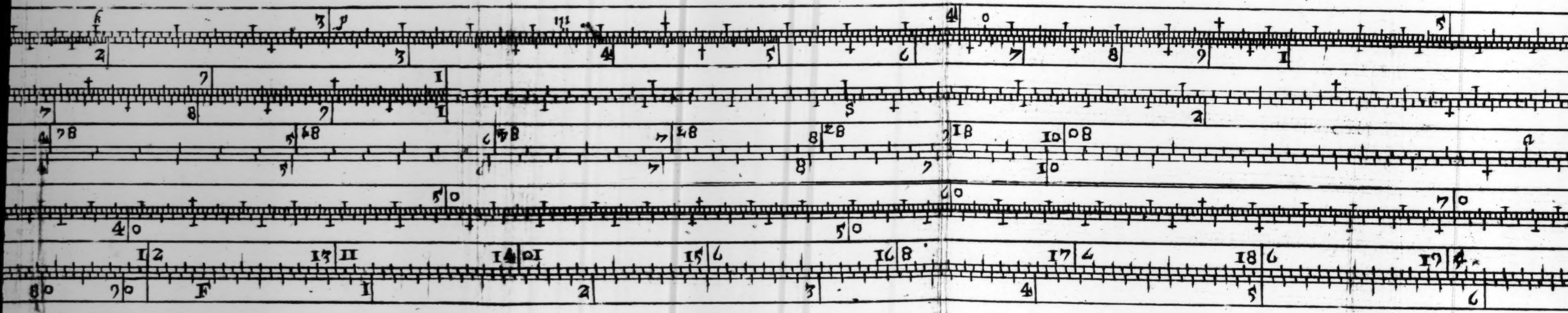
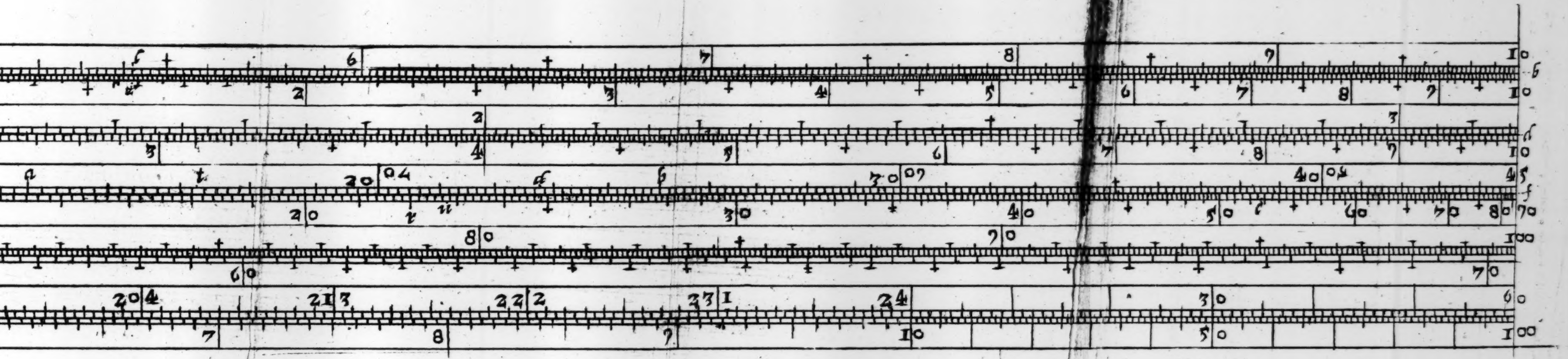




THE USE OF THE Rule of Proportion in Arithmetique and Geometrie.

CAP. I.

The description of the Scales projected upon the Rule of Proportion.

V*Pon the five Lines of the Rule of Proportion, there are ten severall Scales projected,* viz. *two upon each common or middle Line, the one having the divisions thereof shooting downwards, the other upwards:* So the first two Scales meet upon the middle or common Line *a, b*; the next two upon the Line *c, d*,&c.

The uppermost or first Scale of the Rule is a single Line of numbers; first divided into nine unequall parts called *Primes*, and distinguished by the figures 1. 2. 3. 4. 5. 6. 7. 8. 9. And then each of those *Primes* subdivided into ten other parts (according to the same Reason) called *Tenths*: And again each of those *Tenths* subdivided, or at least supposed to be subdivided into ten other parts, as the length of

the Rule will admit: For *example*, upon the scheme of our Rule (hereunto annexed) which is supposed to be about two foot and three inches long between the end-lines, in the foure first *Primes* (*viz.* betwixt the figures 1 and 5) each *Tenth* is really subdivided into ten parts; but in the rest of the *Primes* (*viz.* betwixt the figure 5 and the end of that Scale) each *Tenth* is divided but into five parts; and therefore each of those five parts ought to be esteemed to have the value of 2; and the said tenth parts of those *Tenths* are hereafter called *Centesmes*: Lastly, each of those *Centesmes* is also supposed to be subdivided into ten lesser parts, which are hereafter called *Millaines*: By all which you may observe, that the longer the Rule is, the more small divisions it will admit, and the shorter it is, the fewer.

The second Scale is another Line of Numbers thrice repeated: This Scale shoots upwards upon the Common Line *a*, *b*, and being of a lesser volume then the former, must in some parts thereof content it selfe with lesse divisions, *viz.* from the figure of 5 to the end of that Scale the *Tenths* are onely divided into two parts, and therefore each of those two parts ought to retaine the value of five: All the three parts of this Scale being taken together, are hereafter (for distinctions sake) called the *Little Line of Numbers*, and are in their use distinguished by the first, second, and third part, as they lye in order. They are also of singular use for the ready discovery of the Cube-root, and for the resolution of other necessary operations, as shall be shewed hereafter.

The third Scale is the first Scale repeated, taking beginning from the middle of the Rule, and being broken off at the upper end thereof, is afterwards continued from the lower end of the same to the place where it first beg[an]. This Scale abuts downwards upon the Comm[on] Line *c*, *d*; and the first and this being taken tog[e]ther are hereafter called the *Great Line of Numb*[ers]

wher[e]

whereof the first Scale is called the first part, and this the second.

The fourth Scale is another Line of Numbers twice repeated: This Scale shoots upwards upon the Common Line *c, d,* and being intirely taken together is hereafter called the *Meane Line of Numbers:* It consisteth also of two parts, distinguished by first and second, as they lye in order; and is of necessary use for the finding of the Square-root, and of meane proportions, as shall appeare hereafter.

The fift Scale is a Line of Tangents: This Scale abuts downwards upon the Common Line *e, f,* and doth first containe the Artificiall Tangents of the Quadrant from 0, *degr.* 35. *min.* to 45, *degr.* at the upper end of that Scale, and so (if the Rule would permit) should they be continued forward to 89, *degr.* 25, *min.* but because the divisions of that Scale being inverted will fall out to be the same with the former, they are to be noted and accounted backwards from 45, *degr.* at the upper end of that Scale to 89, *degr.* 25, *min.* at the lower end of the same; each degree thereof being subdivided into six parts, and each of those six parts supposed to containe ten minutes.

The sixt Scale is a Line of Sines: Upon this Scale shooting upwards upon the Common Line *e, f,* are described the Artificiall Sines of the Quadrant from 0, *degr.* 35, *min.* to 90, *degr.* at the upper end of that Scale, each degree (upon our Rule) from 0, *degrees,* 35, *min.* to 30, *degr.* being subdivided into six parts, each part representing ten minutes, as those of the Tangents; but from 30, *degr.* to 50, onely into foure parts, each part containing 15 minutes; from 50 to 70, into two parts, each part comprehending 30 minutes; from 70 to 85, into eaven degrees; and lastly, from 85, *degr.* to 90, not divided at all, but supposed to be divided into five parts, representing those five last degrees of the Quadrant.

The ſeventh Scale ſhooting downwards, is the whole Rule divided into 1000 *equall parts* ; It is hereafter called the *Scale of equall parts*, and is of uſe for the conſtruction and fabrique of the Great Line of *Numbers.*

The eight Scale ſhooting upwards, is a Scale of 70 *degr.* 11 *min. of the Quadrant deſcribed according to* Mercator *and M.* Wrights *projection* : It is hereafter called the *Scale of Latitudes*, and is to be uſed together with the *Scale of equall parts* ; and both of theſe taken together are uſually called the *Meridian Line*, and are of excellent uſe in Navigation, as ſhall be declared hereafter.

The ninth is the Scale of Inch-meaſure, viz. two foot thereof divided into 24 inches, and each inch into ten leſſer parts, counted both forwards and backwards, after the uſuall manner.

The tenth and laſt Scale conſiſts of three ſeverall kinds, viz. a Gage Line, a Line of Cords, and a Scale of Foot-meaſure : The firſt of theſe being ſigned by the letter *G*, is nothing elſe but ſeven inches divided into ten equall parts, and thoſe ſubdivided into ten leſſer parts, and is hereafter to be uſed for the ready diſcovery of the *æquated* diameter (and ſo by conſequent of the content) of any Wine, Beere, or Oyle veſſell : The next marked by the letter *C.* is an ordinary Line of *Cords*, already ſufficiently knowne and of frequent uſe amongſt Artiſts: the third & laſt marked by the letter *F*, is the *Scale of Foot-meaſure* being nothing elſe but a foot firſt divided into ten parts, and thoſe ſubdivided into ten leſſer parts, and ſo (by conſequent) the whole foot ſuppoſed to be thereby divided into 1000 parts.

At the end of theſe two Scales there is another double Scale placed, containing in length three inches *French*, whereof the uppermoſt ſhooting downwards, is a Scale divided into 60 parts, and that ſhooting upwards into 100 parts : The uſe of

these two Scales is for the ready reduction of Sexagenarie minutes to Decimalls, and of Decimall minutes to Sexagenaries, as shall appeare hereafter.

CAP. II.

The Construction and Fabrique of the Lines described upon the Rule of Proportion.

1. TO *describe the* Line of Numbers, *having prepared a Rule of Silver, Brasse, or Wood, (of what length you please) and caused it to be ruled according to the pattern hereunto annexed, and also a Scale of* 1000 *equall parts to be drawne, equall in length to your intended* Line of Numbers, *repaire to the Table of Logarithmes, and therein observing the first four figures of the Logarithme of* 101, *besides the Index or Characteristique (viz.* 0043) *take with your Compasses the distance from the beginning of the said Scale of equall parts to the said* 43 *parts; This done, if you apply that extent of the Compasses upwards from the beginning of the* Line of Numbers, *which you intend to make, the moveable point of the Compasses will fall upon the first* Centesme *of that Line: In like manner by the first foure figures of the Logarithme of* 102, *besides the Index (viz.* 0086) *you may marke the second* Centesme *of the same Line, and so consequently all the rest in their order.*

Example, if it were propounded to make a *Line of Numbers* equall to that of the first Scale, let there be a Scale of equall parts made, equall in length to

that *Line*, such as the seventh Scale before described happens to bee; then extending your Compasses from the beginning of that Scale of equall parts to 0043, *viz.* to the point *v*, apply that extent from the beginning of your Intended *Line of Numbers*; For, that done, the moveable point of the Compasses will fall upon the first *Centesme* of that *Line*, *viz.* at the point *e*: In like manner, the extent from the beginning of the Scale of equall parts to 0086, *viz.* to the point *a* will mark out upon the intended *Line of Numbers* the point *b*, representing the second *Centesme* of that *Line*, and so consequently the rest in order.

2. The *Line of Tangents* is framed much after the same manner; For having before prepared a Scale of equall parts sutable to that *Line*, (*viz.* consisting of halfe the length of the whole Line) *Repaire unto the Table of Artificiall Sines and Tangents, and therein finding the Artificiall Tangent of* 0, *degr.* 40, *min. if (rejecting the Characteristique or first figure thereof) you take off with your Compasses upon your foresaid sutable Scale of equall parts (as before) the foure first figures of the same Tangent (viz.* 0658) *that extent being applyed upwards from the beginning of the* Line of Tangents, *will cause the moveable point of the Compasses to fall upon the division, representing* 0, *degr.* 40, *min. In like manner the extent of* 1627 (*the second, third, fourth, & fift figures of the Tangent of* 0, *degr.* 50, *min.*) *will guide to marke out the same* 0, *degr.* 50, *min. upon the same Line:* And so proceeding you may readily describe all the rest, as they follow in order.

3. The *Line of Sines* may be drawne in all points as the *Line of Tangents*, if you use the second, third, fourth and fift figures of the *Artificiall Sines*, as you are before directed to use those of the *Tangents*. And here *note*, that the Line before called the *Meane Line of Numbers*, and these Lines of *Tangents* and *Sines* are all of them framed by one and the same

Scale,

Scale, and are also hereafter to be used together in the resolution of *Plaine Triangles*, the Scale of equall parts or *Radius*, by which they are made; being in each of them twice repeated.

4. The *Meridian Line* being framed by the ordinary Table of Meridionall degrees, and the making of the *Line of Cords* being obvious to every meane practitioner in the Mathematiques, I shall not need to trouble you with their construction. The other Scales also, which consist of equall parts, will not need any father description.

Cap. III.

Numeration upon the Rule of Proportion.

Probl. I.

A whole number being given, to finde the point where the same is represented upon the Line of Numbers.

Finde amongst the figures, by which the Primes are distinguished, the first figure of the number given; and for the second figure thereof count from the beginning of the Prime, unto which the first figure directs you, so many Tenths as that figure hath Unites; Then for the third figure count from the last Tenth so many Centesmes as that

third figure hath Unites: And so likewise for the fourth figure count from the last Centesme so many Millaines as the same fourth figure hath Unites: This done, you shall at last fall upon the point where the number propounded is represented upon the Line of Numbers.

Example, the number given being 1728, the first figure thereof (*viz.* 1.) leads me unto the first *Prime,* designed by the figure 1, within which *Prime* counting seven *Tenths* for the second figure, and from the seventh *Tenth* two *Centesmes* for the third figure, and from the second *Centesme* eight *Millaines* for the fourth figure; at last I finde the number given to be represented upon the first part of the Great Line of Numbers at the point *h*: So likewise is the number 27 found at the point *k*, the number 542 at the point *l*, and 3345 at the point *m*, &c.

From hence follow these Corollaries:

1. *The figures which any number given hath towards the right hand, besides the first foure figures towards the left hand, are not exprest upon the Rule:* And therefore if the number given were 172845, it would be likewise represented at the point *h*: Howbeit, that uncertainty causeth no inconvenience in the use of the Rule, as shall more plainly appeare hereafter.

2. *The figures by which the Primes are distinguished (in reference to one and the same number) retaine alwayes one and the same value.*

Example, in searching the number 1728, conceiving the figure prefixed at the beginning of the first *Prime* (*viz.* 1.) to have the value of *Thousands,* the figure prefixed before the second *Prime* (*viz.* 2.) ought also to be esteemed to have the value of *Thousands,* and so of the rest in their order: for, according to the same reason that *h* represents 1728, the point *n* wil represent 2000, the point *p* 3000, &c.

3. *The numbers, which have onely the simple value of Unites, as 1. 2. 3. 4. &c. and those which after the*

first

first figure have nothing but cyphers, as 10. 100. 1000. 20. 200. 2000. *&c. are all represented at the same points.*

So 1. 10. 100. 1000. &c. may be al represented at the beginning or end of the *Line:* 2. 20. 200. 2000. &c. at the beginning of the second *Prime:* 3. 30. 300. 3000. &c. at the beginning of the third *Prime*, &c.

4. *The numbers, which being composed of three figures have a cypher in the middle, are found betwixt the beginning of the Prime, unto which they belong, and the first Tenth of the same Prime.*

So 405 beginning by the figure 4 (and therefore to be sought for in the fourth *Prime)* is represented at the point *o*.

5. *The numbers, which being composed of foure figures have two cyphers in the middle, are represented betwixt the beginning of the Prime, unto which they belong, and the first Centesme of the same Prime:* So 1005 is found at the point *q*.

6. *When the Line of Numbers is repeated, and for that cause consisteth of severall parts, the first part thereof is in value a degree lesse than the second, and the second a degree lesse than the third, &c.*

So upon the Meane Line of Numbers, if you conceive 10 at the upper end thereof to represent 100, the figure 1 in the middle (or which is all one, at the beginning of the second part) will represent 10, and 1 at the lower end of that Line (or which is all one, at the beginning of the first part) will represent 1: But if 10 at the upper end thereof shall be conceived to beare but the value of 10, the figure 1 in the middle shall have the value of 1, and 1 at the lower end the value of $\frac{1}{10}$ and 2 the value of $\frac{2}{10}$ &c. In like manner, if 10 at the upper end represent 1, the figure 1 in the middle must represent $\frac{1}{10}$, and 1 at the lower end $\frac{1}{100}$, &c.

PROBL. 2.

To finde a Fraction or broken Number upon the Line of Numbers.

THe Fractions, which are to be found upon the Line of Numbers, ought alwayes to be Decimalls, *viz.* ought alwayes to have for their denominators the figure 1, with nothing but cyphers towards the right hand, ſuch as are $\frac{125}{1000}$ $\frac{25}{100}$ $\frac{5}{10}$ $\frac{75}{100}$ or the like, which may otherwiſe be written thus, .125, .25, .5, .75, and are equivalent to $\frac{1}{8}$ $\frac{1}{4}$ $\frac{1}{2}$ and $\frac{3}{4}$: And therefore if the Fractions propounded be not Decimals, they ought to be reduced to ſuch: For, that done, they may be diſcovered in all points as whole numbers are found out upon the Line, which may be plainly underſtood by the *examples* produced in the ſixt *Corollary* of the laſt Probleme.

PROBL. 3.

To finde a Mixt Number upon the Line of Numbers.

F*Irſt finde by the firſt Probleme aforegoing the point repreſenting the whole parts of the number given, and then afterwards the Fraction or broken parts thereof in the rankes that follow.*

Example, a Line that hath the length of 17 foot and $\frac{28}{100}$ of a foot (which may more conveniently

be

be written thus, 17. 28) being propounded, first I finde the whole parts thereof (*viz.* 17) represented at the point *r*, and after counting two Centesmes, and then eight Millaines, at last I finde the number given to be represented at the point *h*: In like manner if the number propounded were 172.8, or 1.728, it would be still represented at the same point.

PROBL. 4.

Any point of the Line of Numbers being assigned, to finde the figures represented at the same point.

Take the figure prefixed at the beginning of the Prime, within which the point is propounded, for the first of the figures required; then shall the second figure required be composed of so many Unites as there are Tenths intercepted betwixt the beginning of the same Prime and the point given: In like manner shall the third figure required have so many Unites as there are Centesmes comprehended betwixt the last of those Tenths and the said point: And so likewise shall the fourth figure consist of so many Unites as there are Millaines betweene the last Centesme and the point given.

Example, if the point *h* were propounded, because that point is situate within the Prime, before which the figure 1 is prefixed, I take the figure 1 for the first of those required; And then finding seven Tenths betwixt the beginning of that Prime and the point given, I set downe 7 for the second: And so proceeding and finding two Centesmes betwixt the last Tenth and the said point, I take 2 for the third figure: And lastly conceiving eight Millaines to be comprehended betweene the last Centesme

tesme and the point given, I take 8 for the fourth figure required : This done, I conclude, that the figures represented at the point propounded, are 1728. In like manner the point *q* being given, I take 1 for the first figure; but here because I finde no Tenths betwixt the beginning of that Prime and the point given, I write a cypher in the second place; and there also finding no Centesmes I write also a cypher in the third place : And then at last finding the point propounded in the middle of a Centesme (which is supposed to be divided into ten Millaines) I annex in the fourth place 5 : This done, the figures represented at the point given will be found 1005.

Probl. 5.

An Arke or Angle being propounded to finde upon the Rule of Proportion the point which represents the Tangent of the same Arke or Angle.

I*F the Arke or the measure of the Angle exceeds not* 45 *degrees, search the degrees of that Arke or Angle upon the Line of Tangents, mounting upwards from the lower end of that Line towards the upper end of the same.*

So the Tangent of an Arke or Angle, which consists of 15 degrees, is represented at the point *a* : of 25 degrees at the point *b*, &c.

But if the Arke or measure of the Angle exceeds 45 *degrees, looke the degrees thereof, descending downwards from the upper end of the Line towards the lower end of the same.* So the Tangent of 65 degrees is found at the point

point *b*, of 75 degrees at the point *a*, &c.

And if the Arke or Angle propounded (besides the whole degrees) is also composed of certaine minutes, finde first the whole degrees, and after that, betwixt the last degree found and the next that followes, take so many of the parts which may amount to the minutes given, accounting each of the parts contained betwixt the two degrees for ten minutes: So the Tangent of 22 *degr.* 45 *min.* is found at the point *d*, and the Tangent of 72 *degr.* 45 *min.* at the point *t*. And therefore *è converso*, if the points *d* and *t* were given upon this Line, the degrees and minutes represented by them would be 22 *degr.* 45 *min.* and 72 *degr.* 45 *min.* &c.

Probl. 6.

An Arke or Angle being propounded, to finde upon the Rule of Proportion the point, which represents the Sine of the same Arke or Angle.

F*Inde upon the Line of Sines the degrees of the Arke or Angle given, and you have your desire:* So the Sine of the Arke or Angle of 22 *degr.* is represented at the point *r*.

But if the Arke or Angle given have also minutes annexed, first search the whole degrees given, and then betweene that degree found and the next that followes, take so many parts as you have minutes propounded, conceiving the distance betwixt each degree and the next that followes to comprehend 60 minutes.

So the Sine of 22 *degr.* 45 *min.* is found at the point *u*; of 42 *degr.* 50 *min.* at the point *q*; of 52 *degr.* 45 *min.* at the point *c*, &c. And therefore here also

alſo *è converſo*, if the points *u*, *q*, and *c* were aſſigned upon this Line, the degrees and minutes repreſented by them would be 22 *degr.* 45 *min.* 42 *degr.* 50 *min* and 52 *degr.* 45 *min.*&c.

CAP. IV.

The uſe of the Rule of Proportion in Arithmetique.

IN *Arithmetique* there are three ſeverall ſorts of Proportion, *Arithmeticall*, *Geometricall*, and *Muſicall*. *Arithmeticall*, when divers numbers being compared together retaine amongſt themſelves equall differences, as theſe, 2. 4. 6. 8. &c. And this is either *continued*, as in the numbers before produced, or in theſe, 3. 6. 9. 12. 15, &c. which is alſo called *Arithmeticall* Progreſſion, or a ranke of numbers *arithmetically* proportionall: or diſcontinued, as in theſe, 2. 4. 10. 12. or the like. *Geometricall* proportion is, when divers numbers being compared together differ amongſt themſelves according to the ſame rate or reaſon, as theſe, 2. 4. 8. 16. &c. For here, as 2 is half 4, ſo is 4 halfe 8, and 8 halfe 16: this is likewiſe either *continued*, as in thoſe before propounded, or in theſe, 1. 3. 9. 27. 81. &c. or the like, which is alſo called *Geometricall* progreſſion, or a ranke of numbers *geometrically* proportionall: Or *diſcontinued*, as in theſe, 2. 4. 16. 32; for as 4 is double 2, ſo is 32 double 16, but ſo is not 16 being compared with 4. *Muſicall* proportion is that which doth as it were proceed from both the former, as when three numbers or termes being propounded, the firſt beares the ſame proportion to the third, that the difference betwixt

betwixt the firſt and the ſecond beares to the difference betwixt the ſecond and third, as in theſe, 3. 4. *6*. for here, as 3 is halfe *6*, ſo is 1, the difference betwixt 3 and 4, to 2, the difference betwixt 4 and *6*. So likewiſe 2. 3. *6*. and 10. 1*6*. 40. are ſaid to be numbers *muſically* proportionall: For in the firſt of theſe two laſt examples, as 2 is to *6*, ſo is 1 to 3; And in the other, as 10 is to 40, ſo is *6* to 24. Thus have I here thought fit briefly to remember the *Reader* of the ſeverall kindes of Proportion, which he doth uſually finde in the writings of thoſe that treat of *Arithmetique*; to the end that the *Problemes* which follow both in *Arithmetique* and *Geometry* may be the better underſtood.

Probl. I.

Two Numbers being given, to finde a third Geometrically proportionall unto them, and to three a fourth, and to four a fifth, &c.

Extend the Compaſſes upon the Line of Numbers from one of the numbers given to the other; this done, if you apply the ſame extent (upwards or downwards) from either of the numbers propounded, the moveable point of the Compaſſes will fall upon the third proportionall required: And ſo the ſame extent being applyed the ſame way from the third, the moveable point of the Compaſſes will fall upon the fourth proportionall, and from the fourth upon the fifth, &c.

Example, Let it be propounded to finde a third proportionall to theſe two numbers 2 and 4, which may beare the ſame proportion to 4, that 4 beares to

to 2; First, I extend the Compasses upon the first part of the Meane Line of Numbers from 2 to 4; this done, if I apply that extent outright from 4 upwards, the moveable point of the Compasses will fall upon 8 the third proportionall required; and being applyed the same way from 8, the movable point will rest upon 16, the fourth proportionall; and from 16 to 32, the fifth; and from 32 to 64, the sixth proportionall. But now if you would yet continue the Progression farther, and so finde the next proportionall to 64 (because the movable point in that case will fall beyond the Line) apply that extent the same way from 64 in the first part of that Line; which done, the movable point of the Compasses will then fall upon 128, the seventh proportionall; and so proceeding farther you may finde 256, the eighth; 512, the ninth, &c.

Contrariwise, if it were required to finde a third proportionall to the same numbers 2 and 4, which may beare the same proportion to 2, that 2 beares to 4; extend the Compasses upon the second part of the Meane Line of Numbers from 4 to 2 downwards; this done, if you apply that extent from 2 the same way (*viz.* downwards) the movable point will fall upon 1, the third proportionall required; And from 1 upon $\frac{5}{10}$ or ·5, by the last Corollary of the third Chapt. and from ·5 to ·25, by the same Corollary, &c.

In like manner, if the two numbers given were 10 and 9, the Compasses being extended downwards from 10 at the upper end of the same Line of Numbers to 9, and that extent applyed from 9 the same way, the moveable point of the Compasses will rest upon 8 ·1, the third proportionall (for the given numbers being 10 and 9, common sense tells me that it cannot be 81, and therefore ought to bee 8 ·1) and from 8 ·1 the movable point will fall upon

on 7·29, the fourth proportionall, &c. So likewiſe if the numbers propounded were 1 and 9, conceiving 10 at the upper end of the Line to repreſent 1, extend the Compaſſes from thence to 9, which extent being applied downwards from 9, will cauſe the movable point of the Compaſſes to fall upon 81, the third proportionall; and from 81 upon 729, the fourth proportionall; &c. And therefore *note* hence, that 1 at the beginning, 1 in the middle, and 10 at the end of the Line, are all arbitrary points, and may each of them repreſent ſometimes 1, ſometimes 10, ſometimes 100, ſometimes 1000, &c. as the termes by which you are to worke, ſhall require, according to the third Corollary of the third Chapter.

Neverthleleſſe neither do the *examples* before produced, nor thoſe, which ſhall follow in the inſuing Problemes, at all croſſe that which hath beene formerly taught in the ſecond Corollary of the third Chapter: For in the laſt *example*, the end of the Line in regard of the firſt terme given (*viz.* 1) hath the ſingle value of an Unit; but in reſpect of the ſecond terme 9 it challengeth the value of 10; and in reference to the third number 81, the value of 100, &c.

Laſtly, if the numbers given were 10 and 12, the third proportionall upwards would be 14.4, the fourth 17.28, &c. and the numbers 1 and 12 being propounded, the third proportionall upwards (as before) will be 144; the fourth 1728; &c.

The like operations may be alſo performed (and that much more exactly) upon the great Line of Numbers: For *example*, 1 and 4 being given, I deſire to know a third, a fourth, a fifth, &c. geometrically proportionall: To performe this, extend the Compaſſes upon that Line acroſſe from 1 at the beginning of the ſecond part thereof unto 4 upon the firſt part of the ſame; which done, that extent being

ing applyed the same way, (*viz.* upwards and acrosse) will reach from 4 upon the first part unto 16 upon the second, and from thence to 64 upon the first part againe, &c.

PROBL. 2.

One number being given to be multiplied by another number given, to finde the Product.

EXtend the Compasses upon the Line of Numbers from 1 unto the Multiplicator; This done, if you apply that extent the same way from the Multiplicand, the movable point of the Compasses will fall upon the product required.

1. *Example,* Let the Multiplicator given be 25, and the Multiplicand 30: Here if you extend the Compasses upon any of the Lines of Number from 1 unto 25, and then apply that extent the same way from 30, the movable point of the Compasses will fall upon 750, the product required. So 1.728, and 25.6 being propounded to be multiplyed, the product will be found 44.2.

2. *Example,* The two numbers given being 45 and 25, I extend the Compasses upon the second part of the Meane Line of Numbers from 1 to 25; Then (because, if I apply that extent the same way from 45 upon the same part of that Line, the movable point will fall beyond the Line) I apply the same extent the same way from 45 in the first part thereof; which done, the movable point will fall upon 1125, the product desired: So the two numbers given being 1.728, and 64.5, the product required will be 111.4.

3. *Ex-*

3. *Example,* If 75 and 35 were given to be multiplied, the Compasses ought to be extended downwards from 1 to 75 in the first part of the Meane Line of Numbers, or (which is all one) from 10 at the upper end of that Line to 75; for, that extent being applied the same way from 35, will cause the movable point of the Compasses to fall upon 2625 the product required.

4. *Example,* If it were required to finde the content of a piece of ground 8. 75 perches long, and 6. 45 broad; because this question is resolved by multiplying the length by the breadth, I extend the compasses from 10 at the top of the Line to 8. 75; then applying that extent the same way from 6.45, the movable point will fall upon 56. 4, the content required, *viz.* 56 Perches and $\frac{4}{10}$ or .4 of a Perch.

And here you may observe, that these last *examples* and those that are like unto them may likewise be performed in working upwards; But in such cases to shunne too great an extent of the Compasses, it is better to beginne the operation from 10 at the top of the Line, and so to descend downwards according to the instructions before delivered: For (take this for a generall Rule once for all, that) *All operations, which are wrought upon the Rule of Proportion, are best performed, when the legs of the Compasses have the least extension.*

Againe, because this Probleme of *Multiplication,* as also (for the most part) all the rest that follow, are resolved by the finding out of a fourth number *geometrically* proportionall to three other numbers given, we wil therefore here insert this other advertisement: Whensoever question is made of finding a fourth proportionall to three such numbers given, for the better conveniency of working upon the Rule the order of the second and third termes may be changed, so that alwayes care be taken, that the first

first number may still retaine the first place: For *example*, you may say, as 1 is to 25, so is 30 to 750; or as 1 is to 30, so is 25 to 750. And this Rule is diligently to be observed in Multiplication, Division, the Rule of three direct, the resolution of Plane and Sphericall Triangles, and generally in all questions of such like proportions; to the end that in working upon the *Rule of Proportion* we may alwayes avoid too great an extension of the Compasses, and by that meanes performe the worke the more exactly.

Lastly, here observe, that Multiplication, and all other questions hereafter produced, which may be wrought upon the Meane Line of Numbers, may likewise be performed upon the Great Line of Numbers (and that much more exactly) by working either outright or acrosse, as the questions propounded shall require; which (I well hope) I may hereafter leave to the discretion of the ingenious Reader to discover, without any further instruction, they being (indeed) but one and the same *Instrument* represented in differing postures.

PROBL. 3.

A number being propounded to be divided by another number, to finde the Quotient.

EXtend the Compasses upon the Line of Numbers from the divisor to 1; This done, if you apply that extent the same way from the dividend, the movable point will fall upon the number of the Quotient.

1. *Example*, Let 750 be the number given to be divided

divided by 25, the divisor: I extend the Compasses downwards from 25 to 1; then applying that extent the same way from 750, at last the movable point will fall upon 30, the Quotient required.

2. The number 1125 being given to be divided by 25; I extend the Compasses downwards from 25 to 1, then applying that extent the same way from 1125, the movable point will fall upon 45, the Quotient required. The same Quotient will also be found, if changing the termes you first extend the Compasses from 25 to 1125, and then apply that extent from 1; for so also shall the movable point fall upon 45, as before; according to the observation made in the last Probleme: In like manner 111.4 being propounded to be divided by 1.728, the quotient will be found 64.5.

3. The number 2625 being propounded to be divided by 75; extend the Compasses upwards from 75 in the first part of the Meane Line of Numbers to 1, or (which is all one) from 75 in the second part thereof to 10 at the top of the Line; This done, if you apply that extent the same way from 2625, the movable point will from thence reach to 35, the quotient required: So likewise 56.4 being given to be divided by 8.75, the Quotient will be 6.45.

Now to discover of how many figures any quotient ought to consist, it will be necessary to observe how many times the Divisor may be written under the Dividend according to the rules of Division; for, of so many figures shall the Quotient be composed: for *example*, 12231 being given to be divided by 27; because the Divisor 27 may (according to the Rules of Division) be written three times under the Dividend 12231 (as may appeare by this *example*) I say, that the Quotient, which is produced by the division of 12231 by 27 consists of three figures:

12231
27..

For

For, having extended the Compaſſes downwards in the ſecond part of the Meane Line of Numbers from 27 (the Diviſor) to 12231 (the Dividend) and applyed that extent the ſame way from 1, the moveable point will fall in the firſt part upon 453, the Quotient of 12231 divided by 27.

PROBL. 4.

To three numbers given to finde a fourth in a direct proportion.

EXtend the Compaſſes from the firſt number or terme given, unto the ſecond; which done, that extent being applyed the ſame way from the third terme, will cauſe the movable point to fall upon the fourth terme required.

Example, if the circumference of a Circle, whoſe Diameter is 7, be 22; what circumference will a Circle have, whoſe Diameter is 14? Extend the Compaſſes upwards upon the Meane Line of Numbers from 7 in the firſt part thereof, unto 14 in the ſecond; This done, that extent being applyed the ſame way from 22, will make the movable point reſt upon 44, the circumference required.

Or otherwiſe downwards; The circumference of a Circle being 22, and the Diameter thereof 7, how much ſhall the Diameter of a Circle be, whoſe circumference is 44? Extend the Compaſſes downwards from 22 in the ſecond part, to 7 in the firſt; which done, that extent being applied the ſame way from 44, will reach to 14, the Diameter ſought for.

PROB.

PROBL. 5.

To three numbers given to finde a fourth in an inversed proportion.

EXtend the Compasses upon the Line of Numbers from the first of the numbers given to the second, having both the same denomination; this done, if that extent be applyed quite backwards from the third given number, the movable point will fall upon the fourth number you looke for.

Example, if 60 Pioners can make a trench of a certaine length and breadth in 45 houres, how long will it be before 40 men can make such another? Extend the Compasses from 60 to 40 (those termes having both the same denomination, *viz.* of men) This done, that extent being applyed backwards from 45, will reach to 67. 5, the fourth number you looke for; I conclude therefore that 40 men will performe as much in 67 houres and an halfe, as 60 men will doe in 45 houres.

PROBL. 6.

To three numbers given to finde a fourth in a doubled proportion.

THe use of this Probleme appeares chiefly in proportions of Lines to superficies, or of superficies to Lines.

Now

Now if the denomination of the first and second termes be of Lines, *extend the Compasses upon the Line of Numbers from the first terme to the second; this done, that extent being applyed twice the same way from the third terme, will cause the movable point to fall upon the fourth terme required.*

Example, if the content of a Circle whose Diameter is 14 inches, be 154, what will the content of a Circle be, whose Diameter is 28? Here 14 and 28 having the same denomination (*viz.* of Lines) I extend the Compasses from 14 to 28; then applying that extent the same way from 154, the movable point will first fall upon 308, and from thence upon 616, the content desired.

But if the first two termes have the denomination of *areas* or contents, and the *quæsitum* be a Line, this is the Rule: *Extend the Compasses upon the Mean Line of Numbers from the first terme to the second; this done, that extent being applyed the same way upon the Great Line of Numbers from the third terme, will cause the movable point to fall upon the fourth terme required.*

Example, if the Diameter of a Circle, whose *area* is 154, be 14; what Diameter will a Circle have, whose *area* is 616? Extend the Compasses upon the Meane Line of Numbers from 154 to 616; which done, that extent being applyed the same way upon the Great Line of Numbers from 14, will reach to 28, the Diameter required.

PROBL. 7.

To three numbers given to finde a fourth in a tripled proportion.

THe use of this Probleme appeares in the proportion of Lines to Solids, & *contrà.*

If

If therefore the first and second termes have the denomination of Lines, *extend the Compasses upon the Line of Numbers from the first terme to the second; this done, and that extent applyed three times the same way from the third terme, will cause the movable point at last to fall upon the fourth terme required.*

If an Iron Bullet, whose Diameter is 4 inches, weigheth 9 pounds, what is the weight of another Iron Bullet, whose Diameter is 8 inches? Extend the Compasses from 4 to 8; which done, and that extent applyed the same way three times from 9, the movable point will first fall upon 18, then from 18 upon 36, and at last from 36 upon 72, the weight required.

But if the first two termes be weights or contents of Solids, and a Line is sought for: *extend the Compasses upon the Little Line of Numbers from the first terme to the second; This done, and that extent applyed the same way upon the Great Line of Numbers from the third terme, will cause the movable point of the Compasses to fall upon the fourth terme required.*

If the side of a Cube weighing 72 pounds be 8 inches, how many inches is the side of a Cube that weighes 9 pounds? Extend the Compasses downwards upon the Little Line of Numbers from 72 to 9; that done, and the same extent applyed the same way upon the Great Line of Numbers from 8, will cause the movable point to fall upon 4, the side required.

PROBL. 8.

Betwixt two numbers given to finde a meane arithmetically proportionall.

THis Probleme may be performed without the helpe of the *Rule of Proportion*: Neverthelesse, because

because it conduceth to the resolution of the next insuing Probleme, I insert it in this place, and give this Rule for it:

Adde halfe the difference of the given termes to the lesser of them: for, that aggregate is the arithmeticall mean required.

Example, Let 10 and 40 be the termes given: here, if you subtract the one out of the other, their difference will be found 30, whose halfe (15) being added to 10, the lesser terme, their summe (25) is the Arithmeticall meane you looke for.

PROBL. 9.

Betwixt two numbers given, to finde a meane musically proportionall.

Boetius (*Lib.2. Arith. cap.38.*) hath this Rule for it: *Differentiam terminorum in minorem terminum multiplica, & post, junge terminos, & juxta eum, qui inde confectus est, committe illum numerum, qui ex differentiis & termino minore productus est, cujus cum latitudinem inveneris, addas eam minori termino, & quod inde colligitur medium terminum pones.* Multiply the difference of the termes by the lesser terme, and adde likewise the same termes together: this done, if you divide that product by the summe of the termes, and to the quotient thereof adde the lesser terme, that last summe is the musicall meane desired.

Or shorter thus:

Divide the product of the given termes by their summe: for, this done, the quotient doubled is the meane required. So the numbers given being 6 and 12, I say 12 multiplyed

tiplyed by 6 make 72, which divided by 18 (the summe of 12 and 6) leaves 4 in the quotient, whose double (8) is the musicall meane you looke for. This Probleme therefore may be performed by the second and third aforegoing: or yet otherwise thus:

Finde the arithmeticall meane betwixt the numbers given, and then the analogie will be this,

As the arithmeticall meane found is to the greater extreme: so is the lesser extreme to the musicall meane required.

Example, 10 and 40 being propounded, the arithmeticall meane betwixt them (by the last Probleme) is 25: I say then, As 25 is to 40, so is 10 to 16, the musicall meane desired: the terme therefore here sought for may be discovered by the fourth Probleme aforegoing.

And here (I conceive) it will not be amisse to observe, that by this last Rule, having any two numbers propounded, you may interject two other numbers betwixt them, in such sort that they foure being in severall relations compared one with another, may containe in them all the three proportions abovementioned, which kinde of Harmony *Boetius (lib. 2. cap. ult.)* calls *Maxima & perfecta symphonia*: So in the numbers beforementioned 10, 16, 25, and 40, if you compare 10, 25, and 40 together, there shall you finde *Arithmeticall* proportion; if 10, 16, and 40 together, there Harmony, or *Musicall* proportion; if all of them together, there have you *Geometricall* proportion discontinued: For as 10 to 16, so 25 to 40. And this is that *Harmony* which the same *Boetius* (in the same place) affirmeth to have *Magnam vim in Musici modulaminis temperamentis, & in speculatione naturalium quæstionum:* Great force in the composure of Musicke, and in the discovery of the secrets of Nature: And therefore he also averreth in another place (*viz. lib. 1. cap. 2.*) that *the reason of Numbers was the chiefest Rule, accor-*

ding to which Almighty God framed the world: According to that testified of the Wisedome of God (in the Wisedome of *Sal.*cap.11.v.20.) *Thou hast ordered all things in measure, and number, and weight.* The Statists also and Politicians fetch much from these three proportions for the regular direction of a well governed Commonwealth, as may be easily collected out of their writings, and is learnedly proved by *Bodin* in the last chapter of his Commonwealth.

PROBL. 10.

Betwixt two numbers given to finde a meane geometrically proportionall.

Extend the Compasses upon the Meane Line of Numbers from one of the numbers given to the other; this done, and the same extent applyed upon the Great Line of Numbers from either of those numbers towards the other, the movable point will fall in the middle betwixt them, viz. upon the point representing the meane proportionall required.

Example, 8 and 32 being propounded, the meane proportionall betweene them will be found *16*: For if I extend the Compasses upon the Meane Line of Numbers, from 8 in the first part thereof to 32 in the second, and afterwards apply that extent upon the Great Line of Numbers from 8 towards 32, the movable point will fall upon *16*, the meane proportionall demanded; for as 8 is to *16*, so is *16* to 32: so the meane betwixt *6. 4*, and 14. 4, is *9. 6*, &c.

PROB.

Probl. 11.

Betwixt two numbers given, to finde two meanes geometrically proportionall.

Extend the Compaſſes upon the Little Line of Numbers from one of the numbers given to the other: this done, and that extent applyed upon the Great Line of numbers from either of thoſe numbers towards the other, will cauſe the movable point to fall firſt on the third part of the diſtance betweene them, viz. upon the point repreſenting one of the meane numbers required, and being applyed againe the ſame way, will at laſt reſt upon the other proportionall you looke for.

Example, Let 8 and 27 be the two numbers betweene which two meane proportionals are deſired: firſt, I extend the Compaſſes upon the Little Line of Numbers upwards from 8 to 27: then applying that extent twice upon the Great Line of Numbers from 8 towards 27, I finde the movable point to fall firſt upon 12, and then upon 18, which are the two meanes you deſire to know: for as 8 is to 12, ſo is 12 to 18, and 18 to 27.

Probl. 12.

To finde the Square-root of any number under 1000000.

THe *Extraction* of Roots, which is accounted the hardeſt Leſſon in *Arithmetique*, is performed by

the helpe of this *Instrument* with greatest ease and dexterity: for, whereas the *Problemes* before premised, as also those that follow, cannot well be expedited without the joint use of the *Rule* and *Compasses* together, these of the *Extraction* of the *Square* and *Cube* Roots may be resolved onely by *Inspection*, without any trouble at all or ayd of Compasses: so that a man either riding or going in haste may immediately reade upon the *Rule* the root of any *Square* or *Cube* number propounded: which compendious way of *Extraction* cannot choose but prove to be of admirable use, especially in questions that concerne Military Orders, as shall more plainly appeare hereafter. Wherefore to extract the *Square-root* proceed thus:

1. *When the figures of the number given are even, viz. when the number consists of two, foure, or six figures, look the same number in the first part of the Meane Line of Numbers: which done, just at the same point shall you likewise find upon the Great Line of Numbers the Square root you looke for.*

Example, 264196 being propounded, the Square-root thereof will be found 514: for I finde the number 264196 represented in the first part of the Meane Line of Numbers at the point *x*, and at the same point upon the second part of the Great Line of Numbers I observe 514, the Square-root required.

2. *When the figures of the number given are odde, viz. one, three, or five, search the same number in the second part of the Meane Line of Numbers: which done, just at the same point upon the Great Line of Numbers shall you finde also the Square-root demanded.*

Example, 144 being propounded, I demand the Square-root thereof: that number I finde to be represented in the second part of the Meane Line of Numbers at the point *s*, and just there also upon the Great Line of Numbers I discover 12, which is the

the Square-root of the number propounded. So likewiſe is 144 the Square-root of 20736.

PROBL. 13.

To extract the Cube-root of any number under 100000000.

1. *WHen the number propounded conſiſts of one, foure, or ſeven figures, finde it in the firſt part of the Littl Line of Numbers: that done, at the ſame point upon the firſt part of the Great Line of Numbers, you ſhall finde the Cube-root you looke for.*

Example, let the number given be 1728 whereof the Cube-root is required: I finde that number in the firſt part of the Little Line of Numbers at the point *t*, and at the ſame point upon the Great Line of Numbers I alſo diſcover 12, the Cube-root deſired: In like manner is 12.50 the Cube-root of 1950, and 144 the Cube-root of 2985984.

2. *When the number given conſiſts of two, five, or eight figures, ſearch it in the ſecond part of the Little Line of Numbers, and then proceeding as before, you ſhall have your deſire.*

Example, if 14348907 were given, the Root thereof would be found 243: for, that number being found in the ſecond part of the Little Line of Numbers at the point *u*, juſt at the ſame point upon the Great Line I alſo finde 243, the Cube-root required.

3. *When the number propounded conſiſts of three, ſix, or nine figures, looke for it in the third part of the Little Line of Numbers: for ſo likewiſe at the ſame point upon the Great Line will appeare the Root required.*

So the number 159220088 being found in the first part of the Little Line of Numbers at the point 3, his Cube-root is there likewise found upon the Great Line of Numbers to be 542 : And the Cube-root of 159220 is found to be 54.2, &c.

The order of finding out the Cube-numbers upon the severall parts of the Line may be fitly expressed by this Figure.

1	2	3
1	2	3
4	5	6
7	8	9

CAP. V.

The use of the Rule of Proportion in Geometrie, viz.

In the Dimension,

1. Of Plane Triangles.

PROBL. I.

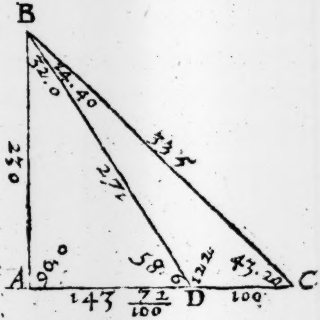
The three Angles and one side being knowne, to finde the other two sides.

TO resolve this Probleme, this is the *Analogie:* As the Sine of the Angle opposed to the side knowne

knowne is, to the parts of the ſame ſide: ſo is the angle oppoſed to one of the ſides unknowne, to the parts which meaſure that ſide: And therefore

Extend the Compaſſes acroſſe from the Sine of the Angle oppoſed to the ſide knowne, to the ſame ſide, found upon the Meane Line of Numbers: then applying that extent the ſame way from the Sine of the Angle oppoſed to one of the ſides required, the movable point will fall upon the parts which meaſure that required ſide.

Example, in the Triangle *C*, *B*, *D*, let the Angle *C* be 43 *degr.* 20 *min.* the Angle *D* 122 *d.* and by conſequent the Angle *B* (being the complement of the two other Angles to 180 *d.* or two right Angles) 14 *degr.* 40 *min.* and let the ſide *D*, *C*, being 100 pace, repreſent the diſtance betweene the two ſtations *D* and *C*: I demand then the diſtance betweene *C* and *B*: Extend the Compaſſes acroſſe from 14 *degr.* 40 *m.* upon the Line of Sines to the middle of the Mean Line of Numbers repreſenting 100; then that extent being applyed the ſame way from 122 *d.* upon the Line of Sines or (which is all one) from 58 *degr.* (for by the Rules of *Trigonometrie* the Sine of an obtuſe Angle and that of his complement to



180 is one and the ſame Line) will cauſe the movable point to fall upon 335, and ſo many paces is the diſtance required: In like manner, the extent being applyed the ſame way from 43 *d.* 20 *m.* upon the Line of Sines, the movable point will fall up-

on 271, the parts of the side *D,B*.

Or otherwise, by changing the termes of the *Analogie*, thus:

Extend the Compasses outright upon the Line of Sines from 14 *d.* 40 *m.* to 58 *d.* then applying that extent the same way upon the Line of Numbers from 100, the movable point will rest upon 335, the distance required: So likewise the Compasses being extended outright upon the Line of Sines from 14 *d.* 40 *m.* to 43 *d.* 20 *m.* and that extent applyed the same way upon the Line of Numbers from 100, the movable point will fall upon 271, the parts of the side *D,B*.

And here observe, that not onely this present Probleme, but also all those that follow (which concern the resolution of Triangles) may be resolved 2 manner of wayes, *viz.* by working either outright, or acrosse, except some few, which we intend to marke in their proper places. Remember likewise what hath beene before touched in the second Chapter aforegoing, *viz.* that the Meane Line of Numbers is the onely Line to be used with these of Sines and Tangents, and no other.

PROBL. 2.

By the knowledge of two sides and an Angle opposed to one of them, to finde the other two Angles and the third side.

THis is the *Inverse* of the last Probleme: for, as the side opposed to the given Angle is, to the Sine of the same Angle: so is the other side

knowne

knowne, to the Sine of the Angle thereunto oppo-
sed: And therefore

Extend the Compasses acrosse from the parts of the side opposed to the Angle knowne, unto the Sine of the same Angle: then that extent being applyed the same way from the parts of the other knowne side, will cause the movable point to fall upon the Sine of the Angle required.

So in the foresaid Triangle *C,B,D*, the side *C,B*, being 335, the Angle *D* (opposed thereunto) 122 *d.*0 *m.* and the side *D, C,* 100, the Angle *B* will be found 14 *d.*40 *m.* For if you extend the Compasses acrosse from 335 upon the Line of Numbers, to 122 *d.*0 *m.* (or rather to 58 *d.*0 *m.* as aforesaid) upon the Line of Sines, and after apply that extent the same way from 100 upon the Line of Numbers, the movable point will rest upon 14 *d.* 40 *m.* the measure of the Angle *B* required.

Now having the knowledge of two Angles, the other may be easily discovered, being the complement of those two to 180, as aforesaid: And the Angles being knowne, the other side may be also found by the Probleme aforegoing.

PROBL. 3.

By the knowledge of two sides and the Angle included, to finde the other two Angles and the third side.

IF the Angle included be a right Angle, this is the *Proportion:* As the greater side is to the lesse so is the Tangent of 45 *d.* 0 *m.* to the Tangent of the lesser Angle: And therefore

Extend

Extend the Compasses upon the Line of Numbers downwards from the greater to the lesse side : then if you apply that extent upon the Line of Tangents the same way from 45 d. *the movable point will fall upon the Tangent of the lesser Angle.*

Example, In the Rectangle triangle *A, B, D*, of the Diagram aforegoing, the side *A, B*, being 230, and the side *A, D*, 143. 72, the Angle *B* will bee found 32 *d.* 0 *m.* For, if you extend the Compasses downwards upon the Line of Numbers from 230 to 143. 72, that extent being applyed the same way from 45 *d.* at the top of the Line of Tangents, will cause the movable point to fall upon 32 *d.* 0 *m. viz.* the measure of the Angle *B*, whose complement 58 *d.* 0 *m.* is the measure of the Angle *D* : And now the three Angles being thus discovered, the third side may also be knowne by the first Probleme of this Chapter.

But if the included Angle be oblique, *viz.* either obtuse or acute, then this is the *Analogie* : As the sum of the sides knowne is, to the difference of the same sides : so is the Tangent of the halfe summe of the Angles unknowne, to the Tangent of halfe their difference : And therefore

Extend the Compasses upon the Line of Numbers downwards, and outright from the summe of the given sides, unto their difference : then applying that extent upon the Line of Tangents from the halfe summe of the angles unknowne, the movable point will fall upon the Tangent of halfe their difference, which being added unto the said halfe sum, makes up the greater, but being deducted from it discovers the lesser of the Angles you looke for.

An *example* of this Probleme, when the moity of the Angles opposed exceeds not 45 *d.*

In the Triangle *B, C, D*, the side *D, B*, being 271, the side *D, C*, 100, and the Angle *D*, 122 *d.* the angle *B* will be found 14 *d.* 40 *m.* and the Angle *C*, 43 *d.* 20 *m.* For, if you extend the Compasses upon the

Meane

Meane Line of Numbers downwards from 371 (the ſum of the ſides knowne) to 171 (their difference) that extent being applyed the ſame way upon the Line of Tangents from 29 *d.* (halfe the ſum of the Angles *B*, and *C*) the movable point will fall upon 14 *d.* 20 *m.* which being added to 29 *d.* amounts to 43 *d.* 20 *m.* for the Angle *C*, and being ſubtracted out of them, the remainder is 14 *d.* 40 *m.* For the Angle *B*.

Two other *examples* of this Probleme, when the moity of the Angles oppoſed exceeds 45 *d.*

1. In the ſame Triangle *C,B,D*, the ſide *C, B*, being 335, the ſide *C, D*, 100, and the Angle *C*, 43 *d.* 20 *m.* the Angle *D* will be 122 *d.* and the Angle *B* 14 *d.* 40 *m.* For if you extend the Compaſſes upon the Line of Numbers downwards from 435 (the ſum of the ſides knowne) to 235 (their difference) that extent being applyed upon the Line of Tangents backwards (*viz.* upwards) from 68 *d.* 20 *m.* (the halfe ſum of the Angles *D* and *B* required) the movable point will fall upon 53 *d.* 40 *m.* which being added to 68 *d.* 20 *m.* their ſum is 122 *d.* 0 *m.* *viz.* the meaſure of the Angle *D*, and being deducted out of the ſame 68 *d.* 20 *m.* the remainder is 14 *d.* 40 *m.* the Angle *B*.

2. The ſide *B, C*, being 335, the ſide *B, D*, 271, and the Angle *B* 14 *d.* 40 *m.* I demand the Angles *D* and *C*: the ſum of the ſides *B, C*, and *B, D*, is 606, their difference is 64, and the Angle *C* being 14 *d.* 40 *m.* the ſum of the Angles oppoſed and unknowne is 165 *d.* 20 *m.* and halfe that is 82 *d.* 40 *m.* Now to ſatisfie this demand, I extend the Compaſſes upon the Line of Numbers downwards from 606 to 64: then, becauſe if I apply that extent upon the Line of Tangents backwards (*viz.* upwards, as before) from 82 *d.* 40 *m.* the movable point will fall as farre beyond the top of that Line, as the terme I looke for is ſituate on this ſide, I apply that extent

down-

downwards from 45 *d.* 0 *m.* causing the movable point also to fall upon the same Line: that done, and the movable point remaining there fixed, I close the Compasses till the other point may rest upon 82 *d.* 40 *m.* And having the Compasses so extended, if applying that extent downwards, I set one of the points at 45 *d.* the other will reach to 39 *d.* 20 *m.* which being added to 82 *d.* 40 *m.* amounts to 122 *d.* *viz.* the Angle *D*: but being deducted out of 82 *d.* 40 *m.* the remainder is 43 *d.* 20 *m.* *viz.* the measure of the Angle *C.*

And in these three cases having discovered the three Angles, the other side may be likewise found by the first Probleme of this Chapter: Observe also that these two last *examples* will not admit of *crosse-worke*: and therefore are exceptions to the generall Rule delivered in the end of the same Probleme.

PROBL. 4.

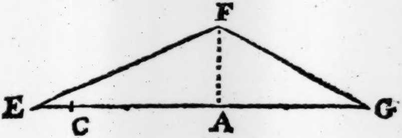
The three sides being knowne, to finde the Perpendicular, and the three Angles.

THe greatest side being assigned for the Base, upon which the Perpendicular shall be supposed to fall, finde the sum and the difference of the other sides: that done, the *proportion* will be this: As the Base is to the sum of the other sides, so is the difference of the other sides to a fourth number, which being deducted out of the Base, the perpendicular wil fall in the middle of that which remains: and therefore

Extend the Compasses upon the Line of Numbers from the parts of the Base unto the summe of the parts of the other sides: this done, and that extent applyed the same

way

way from the difference of the other sides, will cause the movable point to fall upon a fourth number, which if you subtract out of the intire Base, the perpendicular will fall in the middle of the remainder.



Example, in the Triangle *E,F,G*, the side *E,F*, being 13, the side *F,G*, 11, and the Base *E,G*, 20, I demand the point of the Base, where the perpendicular ought to fall, and then the three Angles of the same Triangle: The sum of the sides is 24, and their difference is 2: I extend therefore the Compasses upon the Line of Numbers from 20 to 24: that done, in this *example* (because by the third *Corollary* of the first Probleme of the third Chapter, the numbers 20 & 2 are both represented at the same point) you may observe (without any farther search) the moveable point to discover the parts of the segment *E, C, viz.* 2.4, which being deducted out of 20, there remaines 17.6, whose halfe is 8. 8, which are the parts of the Base comprehended betwixt *C* and *A*, or betwixt *A* and *G*: I conclude therefore that *A* is the point of the Base where the perpendicular ought to fall. Now in the Triangle *A,F,G*, the sides *A,G*, and *G,F*, being known, as also the Angle *F,A,G*, (which is a right Angle by the 10. *Def.* of the 1. *El.* of *Euck.*) the Angles *G*, and *F*, as also the perpendicular *F, A*, may be found by the 1 and 2 Probl. of this Chapter. In like maner in the Triangle *E,F,A*, the sides *E,A* and *E, F*, as also the Angle *E,A,F*, being knowne, the Angles *E*, and *F*, may be found by the 2. Probl.

of

of this Chapter. And laſtly, if you adde the angles *E, F, A*, and *A, F, G*, together, their aggregate will make up the Angle *E, F, G*: And ſo by the knowledge of the three ſides have you all the parts of that Triangle throughly reſolved.

Probl. 5.

The three ſides being knowne, to finde the Area, or ſuperficiall content.

FRom the halfe ſum of the three ſides deduct each ſide, to the end you may diſcover the difference betwixt the ſaid halfe ſum and each ſide: that done, the *Proportions* will be as followeth:

1. *As 1 is to the firſt difference, ſo is the ſecond difference to a fourth number.*

2. *As 1 is to that fourth number, ſo is the third difference to a ſixt number.*

3. *As 1 is to that ſixt number, ſo is the halfe ſumme to an eight number, whoſe Square-root is the Area required.*

Example, the three ſides of the foreſaid Triangle *E, F, G*, being 20, 13, and 11, their ſum is 44, halfe thereof is 22, and the differences betwixt each ſide and that halfe are 2, 9, and 11: The operation being thus prepared (becauſe the number required is a Square-root) I extend the Compaſſes upon the Meane Line of Numbers upwards from 1 to 2: then that extent being applyed the ſame way from 9 (in the firſt part of that Line) the movable point will fall upon 18 the fourth number: this done, and the movable point remaining there fixed, cloſe the Compaſſes, till the other point fall againe upon 1:

for

for that extent being applyed from 11, will cause the movable point to fall upon 198, the sixt number: againe, the movable point remaining there fixed, as before, open the compasses till the other point may yet againe fall upon 1, and may intercept betweene the legs the distance betwixt 1, and 198: for that done, if you apply the same extent (in the first part of the same Line) from 22, the movable point will fall upon 4356, whose Squareroot (by the 12. Probl. of the last Chapter) will appeare at the same point upon the Great Line of Numbers to be *66*, which is also the *Area* required.

2. Of Sphericall Rectangle Triangles.

Probl. 6.

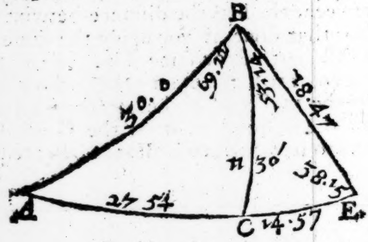
The two sides being given, to finde the Base.

IN Sphericall Rectangle Triangles, the side which subtends the right Angle, is called the *Base*, which to finde by the knowledge of the other sides, use this *Analogie* following:

As the *Radius* or Sine of *90 d.* is to the Sine of the complement (otherwise called the Co-sine) of one of the sides: so is the Co-sine of the other side to the Co-sine of the *Base*: And therefore

Extend

Extend the Compasses downwards upon the Line of Sines from 90 d. to the Co-ſine of one of the ſides: then applying that extent the ſame way from the Co-ſine of the other ſide, the movable point will reſt upon the Co-ſine of the Baſe required.



Example, In the Triangle *A, B, C,* the ſide *A, C,* being 27 *d.* 54 *m.* and the ſide *C, B,* 11 *d.* 30 *m.* the Baſe *B, A,* will be found 30 *d.* 0 *m.* for if you extend the Compaſſes downwards from 90 *d.* to 62 *d.* 6 *m.* (the complement of 27 *d.* 54 *m.*) and after apply that extent the ſame way from 78 *d.* 30 *m.* (the complement of 11 *d.* 30 *m.*) the movable point will fall upon 60 *d.* being the complement of 30 *d.* the *Baſe* required.

PROBL. 7.

The two ſides being knowne, to finde either of the oblique Angles.

As the Sine of the ſide next the Angle required is to the *Radius*: ſo is the Tangent of the oppoſite ſide to the Tangent of the ſame Angle: And therefore,

1. When

1. When the ſide oppoſed to the Angle required exceeds not 45 *d. Extend the Compaſſes upon the Line of Sines from the Sine of the ſide adjacent to the Angle required, to* 90 d. *then that extent being applyed the ſame way upon the Line of Tangents, from the Tangent of the ſide oppoſed to the required Angle, the movable point will fall upon the Tangent of the ſame required Angle.*

1. *Example,* In the ſaid Triangle *A,B,C,* the ſide *A,C,* being 27 *d.* 54 *m.* and the ſide *C,B,* 11 *d.* 30 *m.* I demand the Angle *A.* Extend the Compaſſes upon the Line of Sines from 27 *d.* 54 *m.* to 90 *d.* then that extent being applyed the ſame way upon the Line of Tangents from 11 *d.* 30 *m.* the movable point will reſt upon 23 *d.* 30 *m.* the Angle *A* required.

Or otherwiſe thus: Extend the Compaſſes acroſſe from 27 *d.* 54 *m.* upon the Line of Sines to 11 *d.* 30 *m.* upon the Line of Tangents: then applying that extent the ſame way from 90 *d.* upon the Line of Sines, the movable point will fall upon the Line of Tangents at a point repreſenting 23 *d.* 30 *m.* as before. And *note,* that in this caſe the terme required will alwayes fall out to be leſſe then 45 *d.*

2. *Example,* To know the Angle *B*: Extend the Compaſſes upon the Line of Sines from 11 *d.* 30 *m.* to 90 *d.* then (becauſe that extent being applyed upon the Line of Tangents the ſame way from 27 *d.* 54 *m.* will cauſe the movable point to fall as farre beyond the top of that Line, as the terme required is ſituate on this ſide) apply the ſame extent backwards upon the Line of Tangents from 45 *d.* cauſing the movable point to fall alſo upon the ſame Line: for, that done, and the movable point remaining fixed at the point where it falls, cloſe the Compaſſes till the other point may fall upon 27 *d.* 54 *m.* And at laſt that extent being applyed outright upon the Line of Tangents from 45 *degr.* will cauſe the movable point to reſt upon 69 *d.* 21 *m.* the Angle *B* required. Or otherwiſe: Extend the Compaſſes acroſſe

crosse from 11 *d.* 30 *m.* upon the Line of Sines to 2 *d.* 54 *m.* upon the Line of Tangents: then if you apply that extent backwards from 90 *d.* upon the Line of Sines, the movable point will fall upon the Line of Tangents at a point representing 69 *d.* 21 *m.* as before. And here the required Angle is alwayes greater then 45 *d.*

2. When the side opposed to the Angle required exceeds 45 *d.* *Extend the Compasses upon the Line of Sines from the Sine of the side adjacent to the Angle required, to 90* d. *That done, if you apply that extent backwards upon the Line of Tangents from the Tangent of the side opposed to the said required Angle, the movable point will fall upon the Tangent of the same Angle.*

Example, In the *Diagram* annexed, the side *A, C*, being 61 *d.* 53 *m.* and *B, C*, 54 *d.* 28 *m.* the angle *A* will be found 57 *d.* 47 *m.* For, the Compasses being extended upon the Line of Sines from 61 *d.* 53 *m.* to 90 *d.* and that extent applyed backwards upon the Line of Tangents from 54 *d.* 28 *min.* the movable point will fall upon

57 *d.* 47 *m.* the Angle *A* required. And here *observe*, 1. that in *examples* of this kinde you cannot worke acrosse: 2. the Angle here found is alwayes greater then 45 *d.*

PROBL.

PROBL. 8.

The Base and one of the oblique Angles being given, to finde the other oblique Angle.

As the *Radius* to the Co-sine of the *Base*; so is the Tangent of the Angle knowne to the Co-tangent of the Angle required: And therefore

1. When the Angle given exceeds not 45 *d.* *Extend the Compasses upon the Line of Sines from* 90 d. *to the Co-sine of the Base: then, if you apply that extent the same way upon the Line of Tangents from the Tangent of the Angle given, the movable point will fall upon the Co-tangent of the required Angle.*

Example, In the *Diagram* of the sixt Probl. the Base *A*,*B*, being 30 *d.* and the Angle *A* 23 *d.* 30 *m.* the Angle *B* will be found 69 *d.* 21 *m.* For, if the Compasses be extended upon the Line of Sines from 90 *d.* to 60 *d.* (the complement of the Base) and that extent applyed the same way upon the Line of Tangents from 23 *d.* 30 *m.* the movable point will rest upon 20 *d.* 39 *m.* whose complement (found also at the same point) is 69 *d.* 21 *m.* the Angle *B* required. Or otherwise by crosse-worke, thus: Extend the Compasses from 90 *d.* upon the Line of Sines to 23 *d.* 30 *m.* upon the Line of Tangents: then that extent being applyed the same way from 60 *d.* upon the Line of Sines, the movable point will fall upon the Line of Tangents at the point representing 20 *d.* 39 *m.* as before. And here *observe*, that (in this case) the Angle you look for is alwayes lesse then 45 *d.*

2. When

2. When the Angle given is greater then 45 *d.* *Extend the Compasses upon the Line of Sines from 90 d. to the Co-sine of the Base: this done, if you apply that extent upon the Line of Tangents backwards from the Tangent of the Angle given, the movable point will fall upon the Co-tangent of the Angle required.*

1. *Example,* In the *Diagram* of the sixt Probl. *B, A,* being 30 *d.* and the Angle *B 69 d.* 21 *m.* the Angle *A* will be found 23 *d.* 30 *m.* For if the Compasses be extended upon the Line of Sines from 90 *d.* to 60 *d.* and that extent applyed backwards upon the Line of Tangents from 69 *d.* 21 *m.* the movable point will fall upon 66 *d.* 30 *m.* the complement of 23 *d.* 30 *m.* the Angle *A* required. And in this case you cannot use crosse-worke, and the last terme found upon the Rule is alwayes greater then 45 *d.* but the terme required lesse.

2. *Example,* In the *Diagram* produced in the last Probl. *B, A,* being 74 *d.* 6 *m.* and the Angle *B* 66 *d.* 30 *m.* the Angle *A* will be found 57 *d.* 47 *m.* For, if you extend the Compasses upon the Line of Sines from 90 *d.* to 15 *d.* 54 *m.* and then (because that extent being applyed backwards, as before, upon the Line of Tangents from 66 *d.* 30 *m.* will cause the movable point to fall beyond that Line) if you proceed as you were directed in the second *example* of the said last Probl. at last the movable point will rest upon 32 *d.* 13 *m.* the complement of the Angle *A* required. Or otherwise by crosse-worke: Extend the Compasses from 90 *d.* upon the Line of Sines to 66 *d.* 30 *m.* upon the Line of Tangents: this done, if you apply that extent backwards from 15 *d.* 54 *m.* upon the Line of Sines, the movable point will rest upon the Line of Tangents at the point representing 32 *d.* 13 *m.* as before. And (in this case) the last terme found upon the Rule is alaways lesse then 45 *d.* but the terme required greater.

PROB.

Probl. 9.

The Base and one of the oblique Angles being knowne, to finde the side adjacent to the same Angle.

As the *Radius* is to the Co-sine of the Angle knowne; so is the Tangent of the Base to the Tangent of the side required: And therefore,

1. When the Base is lesse then 45 *d.* *Extend the Compasses upon the Line of Sines from* 90 *d. to the Co-sine of the Angle knowne: then applying that extent the same way upon the Line of Tangents from the Tangent of the Base, the movable point will fall upon the Tangent of the side required.*

So in the *Diagram* of the sixt Probleme, *B*, *A*, being 30 *d.* and *A* 23 *d.* 30 *m.* the side *A*, *C*, (whether you worke outright or acrosse) will be found 27 *d.* 54 *m.* And in this case the terme required is alwayes lesse then 45 *d.*

2. When the Base exceeds 45 *d.* *Extend the Compasses upon the Line of Sines from* 90 d. *to the Co-sine of the Angle knowne, as before: that done, if you apply the same extent upon the Line of Tangents backwards from the Tangent of the Base, the movable point will rest upon the Tangent of the side required.*

So in the *Diagram* produced in the seventh Probleme, *B*, *A*, being 74 *d.* 6 *m.* and the Angle *A* 57 *d.* 47 *m.* the side *A*, *C*, will be found 61 *d.* 53 *m.* And in this case you cannot worke acrosse, and the side to be found will be alwayes greater then 45 *d.*

Now if in applying the extent of the Compasses from the Tangent of the Base, the movable point

falls

falls beyond the Line, worke as you were before di-
rected in the second *example* of the seventh Pro-
bleme aforegoing, and so shall you also in that case
discover the side you looke for, which will then al-
wayes happen to be lesse then 45 *d.*

PROBL. 10.

The Base and one of the oblique Angles being knowne, to finde the side opposed to the same Angle.

As the *Radius* is to the Sine of the *Base*, so is the
Sine of the Angle knowne to the Sine of the
side required: And therefore

Extend the Compasses upon the Line of Sines from 90 d. to the Sine of the Base: for, that extent being applyed the same way from the Sine of the given Angle, will cause the movable point to fall upon the Sine of the side required.

Example, In the *Diagram* of the sixt Probleme;
to know the side *B, C,* extend the Compasses up-
on the Line of Sines from 90 *d.* to 30 *d.* then if you
apply that extent the same way from 23 *d.* 30 *m.*
the movable point will fall upon 11 *d.* 30 *m.* the side
required.

PROB.

PROBL. II.

One of the sides and the oblique Angle next unto it being knowne, to finde the Base.

AS the Co-sine of the Angle knowne is to the *Radius*; so is the Tangent of the side given to the Tangent of the Base: And therefore,

1. When the side given exceeds not 45 *d. Extend the Compasses upon the Line of Sines from the Co-sine of the Angle given, unto* 90 d. *This done, and that extent applyed the same way upon the Line of Tangents from the Tangent of the side given, will cause the movable point to fall upon the Tangent of the Base.* So in the *Diagram* of the sixt Probl. the Angle *A* being 23 *d.* 30 *m.* and the side *A, C,* 27 *d.* 54 *m.* the Base *B, A,* will be found 30 *d.* 0 *m.* But here, if the movable point chance to fall beyond the Line, proceed as you have beene before directed in the second *example* of the 7. Probl. And in that case the terme required will alwayes prove greater then 45 *d.*

2. When the given side exceeds 45 *d. Extend the Compasses upon the Line of Sines from the Co-sine of the Angle given, unto* 90 d. *then, if you apply that extent upon the Line of Tangents backwards from the Tangent of the side given, the movable point will fall upon the Tangent of the Base.* So in the *Diagram* of the seventh Probl. the Angle *A* being 57 *d.* 47 *m.* and the side *A, C,* 61 *d.* 53 *m.* the Base *B, A,* will be found 74 *d.* 6 *m.* And here the terme sought for is alwayes greater then 45 *d.*

falls beyond the Line, worke as you were before directed in the second *example* of the seventh Probleme aforegoing, and so shall you also in that case discover the side you looke for, which will then alwayes happen to be lesse then 45 *d.*

Probl. 10.

The Base and one of the oblique Angles being knowne, to finde the side opposed to the same Angle.

As the *Radius* is to the Sine of the *Base*, so is the Sine of the Angle knowne to the Sine of the side required: And therefore

Extend the Compasses upon the Line of Sines from 90 d. *to the Sine of the Base: for, that extent being applyed the same way from the Sine of the given Angle, will cause the movable point to fall upon the Sine of the side required.*

Example, In the *Diagram* of the sixt Probleme; to know the side *B, C,* extend the Compasses upon the Line of Sines from 90 *d.* to 30 *d.* then if you apply that extent the same way from 23 *d.* 30 *m.* the movable point will fall upon 11 *d.* 30 *m.* the side required.

Prob.

PROBL. II.

One of the ſides and the oblique Angle next unto it being knowne, to finde the Baſe.

AS the Co-ſine of the Angle knowne is to the *Radius*; ſo is the Tangent of the ſide given to the Tangent of the Baſe: And therefore,

1. When the ſide given exceeds not 45 *d. Extend the Compaſſes upon the Line of Sines from the Co-ſine of the Angle given, unto* 90 d. *This done, and that extent applyed the ſame way upon the Line of Tangents from the Tangent of the ſide given, will cauſe the movable point to fall upon the Tangent of the Baſe.* So in the *Diagram* of the ſixt Probl. the Angle *A* being 23 *d.* 30 *m.* and the ſide *A, C,* 27 *d.* 54 *m.* the Baſe *B, A,* will be found 30 *d.* 0 *m.* But here, if the movable point chance to fall beyond the Line, proceed as you have beene before directed in the ſecond *example* of the 7. Probl. And in that caſe the terme required will alwayes prove greater then 45 *d.*

2. When the given ſide exceeds 45 *d. Extend the Compaſſes upon the Line of Sines from the Co-ſine of the Angle given, unto* 90 d. *then, if you apply that extent upon the Line of Tangents backwards from the Tangent of the ſide given, the movable point will fall upon the Tangent of the Baſe.* So in the *Diagram* of the ſeventh Probl. the Angle *A* being 57 *d.* 47 *m.* and the ſide *A, C,* 61 *d.* 53 *m.* the Baſe *B, A,* will be found 74 *d.* 6 *m.* And here the terme ſought for is alwayes greater then 45 *d.*

Probl. 12.

One of the ſides and the oblique Angle next unto it being knowne, to finde the other ſide.

As the *Radius* is to the Sine of the ſide given; ſo is the Tangent of the Angle knowne to the Tangent of the ſide required: And therefore,

1. When the Angle given exceeds not 45 *d. Extend the Compaſſes upon the Line of Sines from* 90 d. *unto the Sine of the given ſide: this done, and that extent applyed the ſame way upon the Line of Tangents from the Tangent of the Angle knowne, will cauſe the movable point to fall upon the Tangent of the ſide required.* So in the *Diagram* of the ſixt Probl. *A, C,* being 27 *d.* 54 *m.* & the Angle *A,* 23 *d.* 30 *m.* the ſide *B, C,* wil be found 11 *d.* 30 *m.* And in *examples* of this kinde croſſe-worke may be uſed, and the terme ſought for is alwayes leſſe then 45 *d.*

2. When the Angle given exceeds 45 *d. Extend the Compaſſes as before: which done, if you apply that extent upon the Line of Tangents backwards from the Tangent of the given Angle, the movable point will fall upon the Tangent of the ſide required.* So in the *Diagram* of the ſeventh Probl. *B, C,* being 54 *d.* 28 *m.* and the Angle *B,* 66 *d.* 30 *m.* the ſide *A, C,* will be found 61 *d.* 53 *m.* This *example* and the like cannot be performed by croſſe-worke; and here the terme found is alwayes greater then 45 *d.* But if in applying the Compaſſes backwards the movable point chance to fall beyond the Line, worke as you were before directed in the ſecond *example* of the ſeventh Probleme

bleme of this Chapter, and then will the terme required be alwayes lesse then 45 *d.*

PROBL. 13.

One of the sides and the oblique Angle next unto it being knowne, to finde the other oblique Angle.

As the *Radius* to the Co-sine of the given side; so is the Sine of the Angle knowne, to the Co-sine of the Angle required: And therefore, *Extend the Compasses upon the Line of Sines from 90 d. to the Co-sine of the side given: this done, that extent being applyed the same way from the Sine of the given Angle, will reach to the Co-sine of the Angle required.* So in the *Diagram* of the sixt Probleme *A, C,* being 27 *d.* 54 *m.* and the Angle *A* 23 *d* 30 *m.* the Angle *B* will be found 69 *d.* 21 *m.*

PROBL. 14.

One of the sides and the Angle opposed unto it being knowne; to finde the Base.

As the Sine of the Angle given is to the Sine of the side given; so is the *Radius* to the Sine of the Base: And therefore

Extend the Compasses from the Sine of the Angle given

ven to the Sine of the given ſide: then if you apply that extent from 90 d. *the movable point will fall upon the Sine of the Baſe.* So in the *Diagram* of the ſixt Probleme, *A*, being 23 *d.* 30 *m.* and the ſide *B, C*, 11 *d.* 30 *m.* the Baſe *B, A*, will be found 30 *d.* 0 *m.*

Probl. 15.

One of the ſides and the Angle oppoſed unto it being knowne, to finde the other oblique Angle.

As the Co-ſine of the ſide given is to the Co-ſine of the Angle given; ſo is the *Radius* to the Sine of the Angle required: And therefore,

Extend the Compaſſes from the Co-ſine of the given ſide, to the Co-ſine of the given Angle: this done, that extent being applyed the ſame way from the Radius, *will cauſe the movable point to fall upon the Sine of the Angle required.* So in the *Diagram* of the ſixt Probleme, the ſide *A, C*, being 27 *d.* 54 *m.* and the Angle *B*, 69 *d.* 21 *m.* the Angle *A*, will be found 23 *d.* 30 *m.*

Probl. 16.

One of the ſides and the Angle oppoſed unto it being knowne, to finde the other ſide.

As the Tangent of the Angle given is to the Tangent of the ſide given; ſo is the *Radius* to the

the Sine of the ſide required: And therefore,

1. When neither the Angle nor ſide given exceeds 45 *d.* *Extend the Compaſſes downwards upon the Line of Tangents from the Tangent of the Angle given, to the Tangent of the ſide given: this done, that extent being applyed the ſame way upon the Line of Sines from* 90 d. *will reach to the Sine of the ſide required.*

So in the *Diagram* of the ſixt Probleme, the Angle *A* being 23 *d.* 30 *m.* and the ſide *B, C,* 11 *d.* 30 *m.* the ſide *A, C,* will be found 27 *d.* 54 *m.*

2. When the Angle and the ſide given doe each of them exceed 45 *d.* *Extend the Compaſſes upon the Line of Tangents upwards from the Tangent of the Angle given to the Tangent of the ſide given: then, if you apply that extent backwards upon the Line of Sines from* 90 d. *the movable point will fall upon the Sine of the ſide required.*

So in the *Diagram* of the ſeventh Probleme, the Angle *B* being 66 *d.* 30 *m.* and the ſide *A, C,* 61 *d.* 53 *m.* the ſide *B, C,* will be found 54 *d.* 28 *m.*

3. When the Angle is greater, and the ſide leſſe then 45 *d.* *Extend the Compaſſes upon the Line of Tangents downwards from* 45 d. *to the Tangent of the Angle given: then, if that extent be applyed the ſame way from the Tangent of the given ſide, the movable point will fall upon a point, which upon the Line of Sines repreſents the Sine of the ſide required.*

So in the *Diagram* of the ſixt Probleme, the Angle *B* being 69 *d.* 21 *m.* and the ſide *A, C,* 27 *d.* 54 *m.* the ſide *B, C,* will be found 11 *d.* 30 *m.* And here obſerve, that *examples* of this kinde may likewiſe be performed by croſſe-worke, the extent of the Compaſſes being applyed backwards: For, having extended the Compaſſes acroſſe from 69 *d.* 21 *m.* upon the Line of Tangents to 90 *d.* upon the Line of Sines, if you apply that extent backwards and acroſſe from 27 *d.* 54 *m.* upon the Line of Tangents, the movable point will fall upon the Sine of 11 *d.* 30 *m.* the ſide required.

 Prob.

Probl. 17.

One of the sides and the Base being knowne, to finde the Angle opposed to the same side.

AS the Sine of the Base is to the *Radius*; so is the Sine of the side knowne to the Sine of the Angle required : And therefore,

If you extend the Compasses from the Sine of the Base unto 90 d. *that extent being applyed the same way, will reach from the Sine of the given side unto the Sine of the Angle required.* So in the *Diagram* of the sixt Probleme, *B, A,* being 30 *d.* and the side *B, C,* 11 *d.* 30 *m.* the Angle *A* will be found 23 *d.* 30 *m.*

Probl. 18.

One of the sides and the Base being knowne, to finde the oblique Angle adjacent unto that side.

AS the Tangent of the Base is to the Tangent of the given side; so is the *Radius* to the Cosine of the Angle required : And therefore,

1. When neither the Base nor the side given exceeds 45 *d.* *the extent from the Tangent of the Base to the Tangent of the side given, being applyed the same way, will reach from* 90 d. *to the Co-sine of the Angle required.*

So

So in the *Diagram* of the sixth Probleme, the Base *B, A*, being 30 *d.* and the side *A, C*, 27 *d.* 54 *m.* the Angle *A* will be found 23 *d.* 30 *m.* And in this case crosse-worke may also be used, if you apply the Compasses the same way they were extended.

2. When the Base and the side given doe each of them exceed 45 *d.* *The extent upwards from the Tangent of the Base to the Tangent of the given side, being applyed backwards, will reach from 90 d. to the Co-sine of the Angle required.*

So in the *Diagram* of the seventh Probleme, the Base *B, A*, being 74 *d.* 6 *m.* and the side *A, C*, 61 *d.* 53 *m.* the Angle *A* will be found 57 *d.* 47 *m.* Howbeit in this case crosse-worke hath no place.

3. When the Base is greater, and the side lesse then 45 *d.* *Worke as you were taught in the third Rule of the sixteenth Probleme aforegoing.*

PROBL. 19.

One of the sides and the Base being knowne, to finde the other side.

As the Co-sine of the side given is to the *Radius*; so is the Co-sine of the Base to the Co-sine of the side required: And therefore,

The extent from the Co-sine of the side given to 90 d. being applyed the same way, will reach from the Co-sine of the Base, to the Co-sine of the side required.

So in the *Diagram* of the sixt Probleme the Base *B, A*, being 30 *d.* and the side *A, C*, 27 *d.* 54 *m.* the side *B, C*, will be found 11 *d.* 30 *m.*

Probl. 20.

The two oblique Angles being knowne, to finde the Base.

AS the Tangent of one of the Angles is to the Co-tangent of the other Angle; so is the *Radius* to the Co-sine of the Base: And therefore,

1. When one of the Angles given, and the complement of the other are each of them lesse then 45 d. *The extent from the Tangent of the Angle lesse then* 45 d. *unto the Co-tangent of the other, will reach from* 90 d. *to the Co-sine of the Base.* So in the *Diagram* of the sixt Probleme the Angle *A* being 23 *d.* 30. *m.* and the Angle *B* 69 *d.* 21 *m.* the Base *B*, *A*, will be found 30 *d.* And here crosse-worke may likewise be used.

2. When one of the Angles is greater, and the complement of the other lesse then 45 *d. Proceed as you have beene taught in the the third Rule of the* 16. *Probleme aforegoing.*

Probl. 21.

The two oblique Angles being knowne, to finde either of the sides.

AS the Sine of one of the Angles is to the Cosine of the other Angle: so is the *Radius* to the Co-sine of the side opposite to the Angle, whose Co-sine was taken: And therefore,

The extent from the Sine of one of the Angles given, to the Co-sine of the other, being applyed the same way, will reach from 90 d. to the Co-sine of the side opposed to the Angle, whose Co-sine was taken.

So in the *Diagram* of the sixt Probleme, the Angle *A* being 23 *d.* 30 *m.* and the Angle *B* 69 *d.* 21 *m.* the side *A, C,* will be found 27 *d.* 54 *m.*

3. Of Sphericall Obliquangle Triangles.

PROBL. 22.

Two Angles and a side opposed to one of them being knowne, to finde the side opposed to the other.

As the Sine of the Angle subtended by the side knowne is to the Sine of the same side; so is the Sine of the Angle subtended by the side required, to the Sine of that side: And therefore,

The extent from the Sine of the Angle opposed to the side knowne, unto the Sine of the same side, being applyed the same way from the Sine of the Angle opposed to the side required, will reach to the Sine of the side so required.

So in the *Diagram* of the sixt Probleme, the Angle E being 38 *d.* 15 *m.* the side *B, A,* 30 *d.* and the Angle *A* 23 *d.* 30 *m.* the side *B, E,* will be found 18 *d.* 47 *m.*

PROBL. 23.

Two sides and the Angle opposed to one of them being knowne, to finde the Angle opposed to the other side.

AS the Sine of the side subtending the Angle knowne is to the Sine of the same Angle; so is the Sine of the side subtending the Angle required, to the Sine of that Angle: And therefore,

The extent from the Sine of the side subtending the Angle knowne, to the Sine of the same Angle, being applyed the same way, will reach from the Sine of the side subtending the Angle required, to the Sine of that Angle.

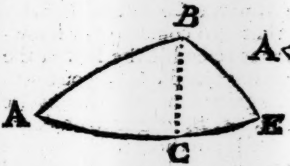
So in the *Diagram* of the sixt Probleme, *B*, *A*. being 30 *d*. the Angle *E* 38. *d* 15 *m*. and the side *B*, *E*, 18 *d*. 47 *m*. the Angle *A* will bee found 23 *d*. 30 *m*.

The studious Reader hath by this time (I presume) so well acquainted himselfe with the turnings and windings of this Instrument, *that in the resolution of most of the insuing Problemes it will (I conceive) be onely necessary to produce the bare* Analogy, *without annexing either Rule or Example, as heretofore, and to referre the proper application thereof to his farther industry and discretion.*

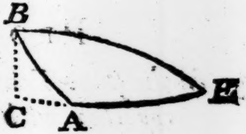
PROB.

PROBL. 24.

In any of the Triangles annexed, the sides A, B, *and* A, E, *together with the Angle* A, *being knowne, to finde the side* B, E.



IN an obliquangle Triangle, when the termes propounded are two sides and one Angle, or two Angles and one side, and yet the terme required undiscoverable by the two last premised Problemes, you are to convert such a Triangle into two Rectangle Triangles, by supposing a perpendicular to be let fall from any one of the Angles upon his opposite side, in such sort that two of the termes propounded may in one of those Rectangle Triangles still remaine given and intire; for by this meanes all the other parts of such a Triangle thus converted, may be readily discovered by the *Analogies* of Rectangle Triangles: And the perpendicular thus imagined, will fall within the Triangle, when the Angles adjacent to the side upon



upon which it falls, are of one and the same kinde that is, both acute, or both obtuse; but otherwise without the Triangle; when those Angles are of differing kindes, *viz.* the one acute and the other obtuse, as plainly appeares by the Triangles annexed, in which (having the sides *A,B*, and *A, E*, as also the Angle *A* propounded) to finde the side *B, E*, use these *Analogies* following:

1. As the *Radius* is to the Co-sine of *A*; so is the Tangent of *A,B*, to the Tangent of *A, C*.

2. As the Co-sine of *A,C*, to the Co-sine of *C,E*; so is the Co-sine of *A,B*, to the Co-sine of *B,E*.

And here observe, that (to come to the knowledge of *C,E*,) in cases that resemble the first of the *Diagrams* annexed, having found *A, C*, you are to deduct it out of *A,E*; againe, in such cases as are like the second *Diagram*, *A, E*, ought to be deducted out of *A,C*; and lastly, in those that resemble the third *Diagram*, *A,C*, and *A, E*, are to be added together.

PROBL. 25.

In the same Triangles, A, B, *and* A, E, *together with the Angle* A, *being knowne, to finde either of the other Angles, and namely (for example) the Angle* E.

1. As the *Radius* to the Co-sine of *A*; so is the Tangent of *A,B*, to the Tangent of *A,C*.

2. As the Sine of *C, E*, to the Sine of *A,C*; so is the Tangent of *A*, to the Tangent of *E*.

PROBL.

PROBL. 26.

A,B, *and* B, E, *together with* A, *being knowne, to finde* A,E.

1. AS the *Radius* to the Co-sine of *A*; so is the Tangent of *A,B*, to the Tangent of *A,C*.

2. As the Co-sine of *A, B*, to the Co-sine of *B, E*; so is the Co-sine of *A, C*, to the Co-sine of *C,E*.

PROBL. 27.

A, B, *and* B, E, *together with* A, *being knowne, to finde* B.

1. AS the *Radius* to the Co-sine of *A,B*; so is the Tangent of *A*, to the Co-tangent of *A,B,C*.

2. As the Tangent of *B, E*, to the Tangent of *A,B*; so is the Co-sine of *A,B,C*, to the Co-sine of *C,B,E*.

PROBL. 28.

A, *and* B, *together with* A, B, *being knowne, to finde either of the other sides, and namely (for example) the side* B, E.

1. AS the *Radius* to the Co-sine of *A,B*; so is the Tangent of *A*, to the Co-tangent of *A,B,C*.

2. As the Co-sine of *C,B,E*, to the Co-sine of *A,B,C*; so is the Tangent of *A, B*, to the Tangent of *B,E*.

PROBL. 29.

A, *and* B, *together with* A, B, *being knowne, to finde* E.

1. AS the *Radius* to the Co-sine of *A,B*; so is the Tangent of *A*, to the Co-tangent of *A,B,C*.

2. As the Sine of *A,B, C*, to the Sine of *C, B,E*; so is the Co-sine of *A*, to the Co-sine of *E*.

PROBL. 30.

A, *and* E, *together with* A, B, *being knowne, to finde* A, E.

1. AS the *Radius* to the Co-sine of *A*; so is the Tangent of *A,B*, to the Tangent of *A,C*.

2. As the Tangent of *E*, to the Tangent of *A*; so is the Sine of *A,C*, to the Sine of *C,E*.

PROB.

PROBL. 31.

A, *and* E, *together with* A, B, *being knowne, to finde* B.

1. AS the *Radius* to the Co-sine of *A, B*: so is the Tangent of *A*, to the Co-tangent of *A, B, C.*

2. As the Co-sine of *A*, to the Co-sine of *E*: so is the Sine of *A, B, C*, to the Sine of *C, B, E.*

PROBL. 32.

Three sides being knowne, to finde any of the Angles.

ADde the three sides together, then from the halfe summe thereof subtract the side opposite to the Angle required: this done, the *proportions* will be as followeth:

1. *As the Radius to the Sine of one of the sides including the Angle required: so is the Sine of the other side including the same Angle, to a fourth Sine.*

2. *As that fourth Sine is to the Sine of the halfe summe of the sides: so is the Sine of the difference betwixt that halfe sum, and the side opposed to the Angle required, to a seventh Sine, betwixt which and 90 d. (at the end of the Line of Sines) if you with your Compasses discover the halfe distance, that point shall represent unto you an Arke, whose complement being doubled is the Angle you looke for.*

So in the *Diagram* of the 6. Probleme the side *A, B*, being

B, being 30 *d.* the side *B,E*, 18 *d.* 47 *m.* and the side *A, E*, 42 *d.* 51 *m.* I demand the Angle *B*: The sum of the sides is 91 *d.* 38 *m.* halfe that sum is 45 *d.* 49 *m.* The side *A,E*, being subtracted out of that halfe, there remaines 2 *d.* 58 *m.* And therefore to discover the Angle *B*, proceed thus:

Extend the Compasses upon the Line of Sines from 90 *d.* unto 30 *d.* then applying that extent the same way, & upon the same Line from 18 *d.* 47 *m.* the movable point will fal upon 9 *d.* 16 *m.* Again, that point remaining there fixed, extend the Compasses so far that their other point may rest upon 45 *d.* 49 *m.* this done, and that extent applyed the same way from 2 *d.* 58 *m.* will cause the movable point at last to fall upon 13 *d* 20 *m.* whose halfe distance towards 90 *d.* will happen upon a point representing 28 *d.* 42 *m.* whose complement (*viz.* 61 *d.* 18 *m.*) being doubled, amounts to 122 *d.* 36 *m.* the quantity of the Angle *B* required.

PROBL. 33.

The three Angles being knowne, to finde any of the sides.

If in stead of the greatest Angle, you take his complement to 180 *d.* the Angles convert themselves into sides, and the sides into Angles, and then (by consequent) the operation will be the same with that of the last *Probleme*.

4. Of

4. Of divers other Geometricall Figures.

PRobl. 34. *The Diameter of a Circle being knowne, to finde the Circumference.*

The extent upon the Line of Numbers from 1 to the Diameter, will reach from 3. 142 to the Circumference.

Probl. 35. *To finde the superficiall content.*

The extent from 1 to the Diameter being twice repeated from .7854 will reach to the Content. Otherwise thus: The extent upon the Great Line of Numbers from 1 to the Diameter, will reach upon the Meane Line of Numbers from .7854 to the Content. Or yet thus: the extent upon the Great Line of Numbers from 1 to .7854 will reach upon the Meane Line of Numbers from the Diameter to the Content. And in this manner may divers of the insuing Problemes be diversified, which (as before) I refer to the discretion of the Practitioner.

Probl. 36. *To finde the side of the square, which may be inscribed within the same Circle.*

The extent from 1 to .7071 will reach from the Diameter to the side of the square required.

Probl. 37. *Having the Circumference, to finde the Diameter.*

The extent from 1 to .3183 will reach from the Circumference to the Diameter.

Probl. 38. *To finde the superficiall Content.*

The extent from 1 to the Circumference being twice repeated from .07958, will reach to the Content. Or, &c. Probl.

Probl. 39. *To finde the side of the square, which may be inscribed within it.*

The extent from 1 to the Circumference, will reach from .2251 to the side of the square required.

Probl. 40. *Having the Content of a Circle, to finde the Diameter.*

The extent from 1 to 1.273 will reach from the Content to another Number, whose square-root is the Diameter required.

Probl. 41. *To finde the Circumference.*

The extent from 1 to 12.57 will reach from the Content to another number, whose square-root is the Circumference required.

Probl. 42. *To finde the side of the square equall unto it.*

Extract the square-root thereof by the 12. *Probl.* of the last Chapter, and you have your desire.

Probl. 43. *The breadth of a long square being given in Inch-measure, and the length in Foot-measure, to finde the Content in feet.*

The extent from 12 to the breadth in inches, will reach from the length in feet to the Content in feet. Or, *vice versa*, the extent from 12 to the length in feet, will reach from the breadth in inches to the Content in feet.

Probl. 44. *The breadth and length of a long square being given in Foot-measure, to finde the Content thereof in yards.*

The extent from 9 to the breadth, will reach from the length to the Content in yards. Or, &c.

Probl. 45. *To finde the Content in single Perches.*

The extent from 16.5 to the breadth, will reach from the length to the content in single Perches. Or, &c.

Probl. 46. *To finde the Content in square-perches, otherwise (in* Architecture*) called* Poles.

The extent from 272.25 to the breadth, will reach from the length to the content in Poles. Or, &c.

Probl.

Probl. 47. *The breadth and length of a long ſquare being given in Perches, to finde the Content in Acres.*

The extent from 160 to the breadth, will reach from the length to the content in Acres. Or,&c.

Probl. 48. *The breadth and depth of a ſquare Rectangle ſolid, being given in Inch-meaſure, and the length in Foot-meaſure, to finde the content thereof in feet.*

The extent from 12 to the breadth or depth in Inches, being twice repeated from the length in feet, will reach to the content in feet. Or,&c.

Probl. 49. *The breadth and depth of a Rectangle ſolid (not juſt ſquare) being knowne in Inch-meaſure, and the length in Foot-meaſure to finde the content in feet.*

Finde (by the tenth *Probleme* of the laſt Chapter) the meane proportionall betwixt the breadth and the depth; for then, the extent from 12 to that Meane Proportionall, being twice repeated from the length in feet, will reach to the content in Feet.

Probl. 50. *The breadth and depth of a Rectangle ſolid (not juſt ſquare) being knowne in Foot-meaſure, to finde the Baſe or ſuperficies at the end thereof.*

The extent from 1 to the breadth, will reach from the depth to the *Baſe* required.

Probl. 51. *The Baſe and length of a Rectangle ſolid being knowne in Foot-meaſure, to finde the content in Feet.*

The extent from 1 to the Baſe, will reach from the length to the content.

Probl. 52. *Having the Diameter of a Cylinder, to finde the Baſe.*

The Baſe of a Cylinder being a perfect Circle, this *Probleme* may be reſolved by the 35. aforegoing.

Probl. 53. *The Baſe and length of a Cylinder being knowne, to finde the content.*

The extent from 1 to the Baſe, will reach from the length to the content.

Probl. 54. *Having the* Axis *of a Sphere, to finde th superficiall content.*

The extent from 1 to the *Axis*, being twice repeated from 3.142, will reach to the superficiall content required. Or,&c.

Probl. 55. *To finde the solid Content.*

The extent from 1 to the *Axis*, being thrice repeated from .5238, will reach to the solid Content required. Or,&c.

Cap. VI.

The use of the Rule of Proportion in Astronomie.

Probl. 1.

By the Sunnes shadow, to finde his height.

THe extent upon the Meane Line of Numbers, from the length of the Rules shadow to the height thereof (held perpendicular to the Horizon) will reach upon the Line of Tangents from 45 *d.* to the Sunnes height required.

Probl. 2. *The Sunnes greatest declination, together with his distance from the next Equinoctiall point being knowne, to finde his present declination.*

As the *Radius* to the Sine of the Sunnes distance from

from the next Equinoctiall point; so is the Sine of the Sunnes greatest declination to the Sine of the declination required.

Probl. 3. *To finde the right ascension.*

As the *Radius* to the Tangent of his distance, &c. so is the Co-sine of his greatest declination to the Tangent of his right Ascension.

Probl. 4. *The Sunnes greatest declination, together with his present declination, being knowne, to finde his right Ascension.*

As the Tangent of his greatest declination to the *Radius*, so is the Tangent of his present declination to the Sine of his right ascension.

Probl. 5. *The elevation of the Pole, together with the Suns declination being knowne, to finde how long the Sun riseth or setteth before or after the houre of six.*

As the co-tangent of the Elevation is to the *Radius*; so is the Tangent of the Sunnes declination to the Sine of the *ascensionall* difference betweene the houre of six, and the Sunnes rising or setting.

Probl. 6. *To finde the Sunnes amplitude.*

As the Co-sine of the Elevation is to the Sine of the Declination; so is the *Radius* to the Sine of the Amplitude.

Probl. 7. *The Elevation of the Pole, the Sunnes greatest declination, and his distance from the next equinoctiall point being knowne, to finde the Amplitude.*

As the Co-sine of the Elevation is to the Sine of the Sunnes distance; so is the Sine of the Sunnes greatest declination to the Amplitude required.

Probl. 8. *When the Sunne is in the Equinoctiall, by knowing the elevation of the Pole, to finde the Suns height at any time assigned.*

As the *Radius* to the Co-sine of the Elevation; so is the Sine of the Sunnes distance from six a clock to the Sine of the height required.

Probl. 9. *The Elevation of the Pole, and the declination of the Sun being knowne, to finde the Suns height at the houre of six.*

As

As the *Radius* to the Sine of the Latitude; so is the Sine of the declination to the Sine of the height required.

Probl. 10. *To finde the Sunnes height at any time assigned.*

1. As the Radius to the Co-tangent of the Elevation; so is the Sine of the Sunnes distance from six, to the Tangent of an Arke, which being subtracted out of the Sunnes distance from the Pole, I say againe,

2. As the Co-sine of the Arke found is to the Co-sine of the residue of the Sunnes distance from the Pole; so is the Sine of the Elevation to the Sine of the height required.

Probl. 11. *To finde the time when the Sun will be due East and West.*

As the Tangent of the Elevation to the *Radius*; so is the Tangent of the Declination to the Co-sine of the houre from the Meridian.

Probl. 12. *To finde the Suns height, when he commeth to be due East and West.*

As the Sine of the Elevation to the *Radius*; so is the Sine of the declination to the height required.

Probl. 13. *To finde the Suns Azimuth at the houre of six.*

As the Co-sine of the Elevation is to the Co-tangent of the Declination; so is the *Radius* to the Tangent of the Azimuth from the North part of the Meridian.

Probl. 14. *The complement of Elevation, the Sunnes distance from the Pole, and the complement of the Suns height being knowne, to finde the Azimuth.*

Having added the three given termes together, finde the difference betwixt their halfe summe and the Sunnes distance from the Pole: this done, the *Proportions* will be as followeth:

1. As the *Radius* to the Co-sine of the Elevation; so is the Co-sine of the height to a fourth Sine:

2. As

2. As that fourth Sine is to the Sine of the halfe summe; so is the Sine of the difference to a seventh Sine, whose halfe distance towards 90 *d.* will discover the Sine of an Arke, whose complement being doubled is the *Azimuth* you look for.

Probl. 15. *To finde the houre of the day.*

Having added the three given termes together, as before, finde the difference betwixt their halfe summe and the complement of the Suns height: this done, the *proportions* will be these:

1. As the *Radius* to the Co-sine of the Elevation; so is the Sine of the Suns distance from the Pole to a fourth Sine.

2. As that fourth Sine is to the Sine of the halfe summe: so is the Sine of the difference to a seventh Sine, whose half distance towards 90 *d.* will discover the Sine of an Arke, whose complement being doubled and converted into time, will produce the houre required.

Cap. VII.

The use of the Rule of Proportion in Dialling.

Probl. 1. *To make a direct Polar Diall.*

Having assigned a Line drawne in the middle of the Plane for the Meridian, and another Line

Line drawne parallel unto it for some other houre, which may be described upon the Plane : I say.

1. As the Tangent of that houre is to the *Radius*; so is the distance of that houre-line from the Meridian to the height of the stile.

2. As the *Radius* is to the height of the stile ; so is the Tangent of any other houre, to the distance of the same houre from the substile.

Probl. 2. *A Meridian Diall.*

Having drawne a Line representing part of the *Axis* of the world towards a proper side of the Plane, (according to his situation either Eastward or Westward) and assigned that Line for the houre of six, the *Proportions* will fall out to be as in the former *Probleme*; for,

1. As the Tangent of any houres distance from six is to the *Radius*; so is the distance of the houre upon the Plane from the Houre-line of six, to the height of the stile.

2. As the *Radius* is to the height of the stile : so is the Tangent of any other houres distance from six, to the distance of the same houre from the substile.

Probl. 3. *An horizontall Diall.*

As the *Radius* to the Tangent of the hour given : so is the Sine of the Elevation to the Tangent of the hour-line from the Meridian.

Probl. 4. *A verticall Diall.*

As the *Radius* to the Tangent of the houre : so is the Co-sine of the Elevation to the Tangent of the Houre-line from the Meridian.

Probl. 5. *A verticall Inclining Diall.*

Having found out the Elevation of the Pole above the Plane, according to its inclination, the Proportion will be this :

As the *Radius* to the Tangent of the Houre : so is the Sine of the Elevation above the Plane, to the Tangent of the Houre-line from the Meridian.

Probl.

Probl. 6. *A verticall Declining Dyall.*

1. As the *Radius* to the Co-tangent of the Elevation: so is the Sine of the Declination to the Tangent of the Substiles distance from the Meridian of the Place.

2. As the *Radius* to the Co-sine of the Declination: so is the Co-sine of the Elevation to the Sine of the Stiles height above the Substile.

3. As the Sine of the Elevation is to the *Radius*: so is the Tangent of the Declination to the Tangent of the Inclination of the Meridian of the Plane to the Meridian of the Place.

4. As the *Radius* to the Sine of the Stiles height above the Substile: so is the Tangent of the Angle at the Pole comprehended betweene the houre given and the Meridian of the Plane, to the Tangent of the Houre-lines distance from the Substile.

Probl. 7. *A Meridian Inclining Dyall.*

1. As the *Radius* to the Tangent of the Elevation: so is the Sine of the Inclination to the Tangent of the Substiles distance from the Meridian.

2. As the *Radius* is to the Sine of the Elevation: so is the Co-sine of the Inclination to the Sine of the stiles height above the Substile.

3. As the Co-sine of the Elevation is to the *Radius*: so is the Tangent of the Inclination, to the Tangent of the Inclination of Meridians.

4. As the *Radius* is to the Sine of the Stiles height above the Substile: so is the Tangent of the Angle at the Pole, to the Tangent of the Houre-lines distance from the Substile.

Probl. 8. *A Polar Declining Dyall.*

1. As the *Radius* to the Sine of the Declination: so is the Co-sine of the Elevation to the Co-sine of the Arke comprehended between the Horizon and the Substile.

2. As the *Radius* to the Tangent of the Declination: so is the Sine of the Elevation to the Tan-

 gent

gent of the Inclination of Meridians, which being converted into time, ſheweth how many houres the Subſtile ought to be placed from the Houre-line of 12.

3. As the *Radius* is to the Tangent of the houres diſtance from the Subſtile: ſo are the parts of the height of the Stile, to the diſtance of the Subſtile from the Houre-line required, meaſured by a Scale of like parts.

Probl. 9. *A Declining inclining Dyall.*

1. As the *Radius* to the Tangent of Inclination to the Horizon: ſo is the Co-ſine of Declination to the Tangent of the Arke of the Meridian of the Place intercepted betweene the Horizon and the Plane, which being compared with the Elevation of the Pole, the diſtance of the Pole from the Plane may be thereby readily diſcovered.

2. As the *Radius* is to the Sine of Declination from the Verticall: ſo is the Sine of Inclination to the Horizon, to the Co-ſine of the Inclination to the Meridian.

3. As the *Radius* is to the Co-ſine of Inclination to the Horizon: ſo is the Co-tangent of Declination to the Tangent of the Arke of the Plane intercepted betweene the Horizon and the Meridian of the Place.

4. As the *Radius* is to the Sine of the Inclination to the Meridian: ſo is the Tangent of the Poles diſtance from the Plane, to the Tangent of the Subſtiles diſtance from the Meridian.

5. As the *Radius* is to the Poles diſtance from the Plane: ſo is the Sine of the Inclination to the Meridian, to the Sine of the Stiles height above the Subſtile.

6. As the Co-ſine of the Poles diſtance from the Plane is to the *Radius*: ſo is the Co-tangent of the Inclination to the Meridian, to the Tangent of the Inclination of Meridians.

7. As

7. As the *Radius* is to the Stiles height above the Subſtile : ſo is the Tangent of the Angle at the Pole, to the Tangent of the Houre-lines diſtance from the Subſtile.

Cap. VIII.

The uſe of the Rule of Proportion in Geographie.

Probl. 1. *Two places being propounded, which differ onely in Latitude, to finde their diſtance.*

1. WHen the two places are ſituate under the ſame Meridian, and upon the ſame ſide of the Equinoctiall; *Subtract the leſſer Latitude out of the greater; that done, the remainder is the diſtance required.*

2. When one of the places propounded is ſituate upon this ſide the Equinoctiall, and the other upon that, and yet both under the ſame Meridian, as before: *Adde the two Latitudes together; this done, their ſum is the diſtance required.*

Probl. 2. *Two places, which differ onely in Longitude, being propounded, to know their diſtance.*

1. When the Places are both of them ſituate under the Equinoctiall: *Subtract the leſſer longitude out of the greater: this done, the remainder is the diſtance required.*

2. When the places are ſituate under ſome parallel betwixt the Equinoctiall and one of the Poles: *Then, as the Radius is to the Co-ſine of the common Latitude given: ſo is the Sine of halfe the difference of Longitude to the Sine of halfe the diſtance.*

Probl.3. *Two places being given, which differ both in Longitude and Latitude, to finde their diſtance.*

1. When one of the places is ſituate under the Equinoctiall, and the other towards one of the Poles: Then, *As the Radius is to the Co-ſine of the difference of Longitude: ſo is the Co-ſine of the Latitude given, to the Co-ſine of the diſtance required.*

2. When both places are without the Equinoctiall, and towards one of the Poles: Then, *As the Radius is to the Co-ſine of the difference of Longitude: ſo is the Co-tangent of the leſſer Latitude to the Tangent of another Arke,* which being ſubtracted out of the complement of the leſſer Latitude, retaine the Ark thereof remaining, and ſay againe, *As the Co-ſine of the Arke found is to the Co-ſine of the Arke remaining: ſo is the Sine of the leſſer Latitude to the Co-ſine of the diſtance required.*

3. When both places are without the Equinoctiall, and one of them ſituate towards the North Pole, and the other towards the South: ſay thus, *As the Radius is to the Co-ſine of the difference of Longitude: ſo is the Co-tangent of one of the Latitudes, to the Tangent of another Arke,* which being ſubtracted out of the other Latitude, and 90 *d.* added together: ſay again, *As the Co-ſine of the Arke found is to the Co-ſine of the Arke remaining: ſo is the Sine of the Latitude firſt taken, to the Co-ſine of the diſtance required.*

CAP.

Cap. IX.

The use of the Rule of Proportion in Navigation.

Probl. 1. *The Latitudes of two places being knowne, to finde the Meridionall difference.*

1. WHen one of the places is situate under the Equinoctiall, and the other without: *The degrees and decimall Minutes found upon the Scale of Equall parts at the point, where that other Latitude is represented upon the Scale of Latitudes, are the Meridionall difference required.*

2. When one of the places have Southerly, and the other Northerly Latitude: *Extend the Compasses upon the Line of Latitudes, from the beginning of that Line to the lesser Latitude: that done, if you apply that extent upon the same Line, and the same way from the greater Latitude, the movable point will discover upon the Line of Equall parts, the Meridionall difference desired.*

2. When both places have Northerly or Southerly Latitude: *Extend the Compasses upon the Line of Latitudes from one of the Latitudes to the other: this*

done, if you apply that extent from the beginning of the Line, the movable point will shew you upon the Scale of Equall parts the Meridionall difference you looke for.

Probl. 2. *The Latitudes of two places together with their difference of Longitude being knowne, to finde the Rumbe directing from the one to the other.*

As the Meridionall difference is to the difference of Longitude: so is the *Radius* to the Tangent of the Rumbe: And therefore,

The extent upon the Meane Line of Numbers from the Meridionall difference to the difference of Longitude, will reach upon the Line of Tangents from 45 d. *to the Tangent of the Rumbe.*

And *note* here, that in this Probleme and the like, you may make use of the double Scale, placed upon the last Line of the Rule of Proportion, at the end of the Scale of Inches: *viz.* (if need be) for the more speedy reduction of the Sexagenary Minutes of the Longitude into Decimalls, *& contrà:* to the end you may by that meanes the more readily work by them upon the Meane Line of Numbers.

Probl. 3. *By both Latitudes and Rumbe to finde the distance upon the Rumbe.*

As the Co-sine of the Rumbe to the true difference of Latitudes: so is the *Radius* to the distance required: And therefore,

Extend the Compasses acrosse from the Co-sine of the Rumb (found upon the Line of Sines) to the true difference of Latitudes (found upon the Meane Line of Numbers:) this done, if you apply that extent the same way and acrosse from 90 d. *upon the Line of Sines, the movable point will shew you upon the Meane Line of Numbers (in Degrees and Decimall Minutes) the distance required.*

Probl. 4. *By both Latitudes and Rumbe, to finde the difference of Longitude.*

As the *Radius* to the Tangent of the Rumbe: so is the Meridionall difference of the Latitudes

to

to the difference of longitude required: And therfore

The extent upon the Line of Tangents from 45 d. *to the Tangent of the Rumbe, will reach upon the Meane Line of Numbers from the Meridionall difference of the Latitudes to the difference of Longitude required.*

Probl 5. *By both Latitudes and distance to finde the Rumbe.*

As the distance is to the true difference of Latitudes: so is the *Radius* to the Co-sine of the Rumb: And therefore,

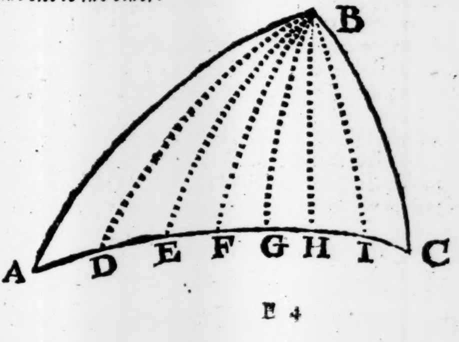
The extent upon the Meane Line of Numbers, from the distance to the difference of Latitudes, will reach upon the Line of Sines from 90 d. *to the Co-sine of the Rumb.*

Probl. 6. *By one Latitude, Distance, and Rumbe, to finde the other Latitude.*

As the *Radius* to the Co-sine of the Rumb: so is the distance to the true difference of Latitudes: And therefore,

The extent upon the Line of Sines from 90 d. *to the Co-sine of the Rumbe, will reach upon the Meane Line of Numbers, from the distance to the true difference of Latitudes.*

Probl. 7. *The Latitudes and difference of Longitude of two places being knowne, to saile by the great circle from the one to the other.*



In the Triangle *A*, *B*, *C*, let *A* represent *S. Christophers*, *C*, the *Lizard*, *B*, the North Pole, *A*, *B*, the complement of the Latitude of *S. Christophers*, *viz.* 74 *d.* 30 *m.* *B*, *C*, the complement of the Latitude of the *Lizard*, 40 *d.* 0 *m.* and *A*, *B*, *C*, the difference of Longitude, 68 *d.* 30 *m.* Now therefore to steere a course from *A* to *C* alongst the Arke *A*, *C*, proceed thus:

1. By the 24 and 25 *Problemes* of the fift Chapter finde the side *A*, *C*, as also the Angles *A*, and *C*.

2. By the 22 of the same Chapter finde the perpendicular *B*, *I*, cutting the side *A*, *C*, at right Angles.

3. By the 8 of the same discover the Angle *A*, *B*, *I*, and by the 9 the side *A*, *I*.

4. Lessening the Angle *A*, *B*, *I*, two, five, or ten degrees, as you shall see cause, (for *example*, by the Angle *A*, *B*, *d*,) by the knowledge of the Angle *d*, *B*, *I*, and of the side *B*, *I*, finde by the 11, 12 and 13 *Problemes* of the same 5 Chapter, the Base *B*, *d*, the side *d*, *I*, and the Angle *B*, *d*, *I*; and so proceeding to doe the like at the points *e*, *f*, *g*, and *h*, you may thereby discover the severall distances betwixt point and point, the severall Latitudes at those points, and the severall Angles, according to which you are to direct your course: For at first, from *A* you are to steere according to the Angle *B*, *A*, *I*, untill you shall have sailed so many Leagues as answer to the distance betwixt *A* and *d*: and then from *d*, according to the Angle *B*, *d*, *I*, untill you shall arrive at the point *e*, according to the number of Leagues that *d* and *e* are distant the one from the other: and so consequently of the rest in their order, untill you shall attaine the point *I*, from whence you are to steere full West towards *C*, the Angle *B*, *I*, *C*, being a right Angle, &c.

CAP.

Cap. X.

The use of the Rule of Proportion in the gaging of Vessell.

Probl. 1. *The true content of a solid Measure being knowne, to finde the Gage-point of the same Measure.*

THe Gage-point of a solid Measure is the Diameter of a Circle, whose superficiall content is equall to the solid content of the same Measure: so the solid content of a Wine-gallon (according to *Winchester* measure) being 231 Cube-inches, if you conceive a Circle to containe so many inches, you shall finde (by the 40 *Probleme* of the 5 Chapter) the *Diameter* thereof to be 17.15: For,

As 1 *is to* 1.273 *: so is* 231 *to* 294.1, *whose Square-root* (by the 12 *Probleme* of the same Chapter) *is* 17.15, *the Gage-point of Wine-measure.*

Thus likewise may you easily discover the Gage-point of Ale-measure, an Ale-gallon (as it hath beene of late discovered) containing 288 Cube-inches: For,

As 1 *is to* 1.273 *: so is* 288 *to* 366.7, *whose Square-root is* 19.15, *the Gage-point of Ale measure.*

And (indeed) 288 Cube-inches seeme to be the most probable content of an Ale-gallon, being the sixt part of 1728, which is the number of Cube-inches contained in a Cube-foot. For so (according to that account) a Cube-foot containes just six Gallons, and the Gage-point of Ale-measure (by reason of the soile and waste) exceeds that of Wine-measure just two Inches.

After the same manner also may you discover the Gage-point of any foraine Measure whatsoever, and afterwards by that meanes come to the knowledge of the true content of their Vessell, according to the Measures used amongst them, as will plainly appeare by that which shall hereafter bee taught for the discovery of the contents of Wine and Beer-vessell according to the English Measures.

Now from that which is abovesaid doth necessarily follow this Corollary: *When the Diameter of a Cylinder in Inches is equall to the Gage-point of any measure (given likewise in Inches) every Inch in the length thereof containes one Integer of the same measure:* So in a Cylinder having 17. 15 Inches diameter, every Inch in the length thereof containes one intire Wine-gallon: and in another having 19.15 inches diameter, every Inch thereof containes one Ale-gallon, &c.

Probl. 2. *In a Wine or Beere vessell, the Diameters at the Head and Bongue being knowne, to finde the equated Diameter.*

Extend the Compasses upon the *Line of Inches* from the Diameter at the Head, to the Diameter at the Bongue: then applying that extent from the beginning of the same Line, and observing there the difference betwixt the two Diameters, (one of the points remaining still fixed at the beginning of that Line) close the Compasses till the other point may fall upon so many parts of the *Gage-line*, as the difference betweene the two Diameters amounts unto

in

in Inches: this done, and that extent applyed from the Diameter at the head towards the Diameter at the Bongue, will cauſe the movable point to fall upon the *equated* Diameter you look for.

Example, The Diameter at the head being 18. 3 Inches, and that at the Bongue 21. 5 Inches, I demand the *equated* Diameter. Firſt, extending the Compaſſes upon the Line of Inches from 18. 3 Inches to 21. 5, and then applying that extent from the beginning of the ſame Line, I finde the movable point to fall upon 3. 2 Inches, *viz.* the true difference of the two Diameters: Now therefore if ſtill keeping one of the points of the Compaſſes fixed at the beginning of that Line, I cloſe them til the other point may fal at 3. 2 upon the *Gage line*, & after apply that extent from 18. 3 (the Diameter at the Head) the movable point will at laſt fall upon 20. 54 Inches, the *equated* Diameter required. And by this meanes your Veſſell, which before was in part of an Ovall forme and irregular, is now reduced into a perfect *Cylinder.*

Probl. 3. *The equated Diameter and length of a Wine or Beere veſſell being given in Inches, to finde the content thereof in Wine-meaſure.*

The extent upon the *Line of Numbers* from 17. 15 (the Gage-point of Wine-meaſure) to the *equated* Diameter, being twice repeated from the length, will reach to the content in Wine-gallons.

Probl. 4. *To finde the Content in Ale-meaſure.*

The extent from 19. 15 (the Gage-point of Ale-meaſure) to the *equated* Diameter, being twice repeated from the length, will reach to the content in Ale-gallons.

Probl. 5. *Having the length and the two Diameters at the Head and Bongue, together with the equated Diameter and Content of a Veſſell, out of which ſo much and no more of the liquor is drawne, that the ſuperficies thereof may cut ſome part of the head; to finde the true quantity of the remainder.*

Deduct

Deduct halfe the difference of the Diameters at the Head and Bongue, out of the distance intercepted between the Bongue & the superficies of the liquor, to the end you may thereby discover where the liquor within the Vessell cuts the head, according to which draw a Line with Chalke (or otherwise) upon the Head, then having drawne another Line parallel to the first, and of like distance from the other opposite side of the Head, you have in the middle of the Head betwixt those two Lines a Segment of the Vessell marked out, and likewise two other Segments, the one above and the other below that middle Segment : after this, taking the length of one of those parallels in Inch-measure, the *equated* Diameter of the superficies may be thus found out upon the *Rule :*

The extent from the Diameter at the Head to the equated Diameter of the Vessell, will reach from the length of one of the Parallels to the equated Diameter of the superficies.

Then having discovered (by the 2 *Probleme* aforegoing) the *equated* Diameter of those two other *equated* Diameters, finde (by the 10 *Probleme* of the 4 Chapter) the meane proportionall between that third *equated* Diameter and the distance betwixt the two Parallels : This done, make use of that meane proportionall, as an *equated* Diameter of the middle Segment, and then finding by one of the two last *Problemes* (according to the question propounded) the content thereof in Gallons, &c. deduct that content out of the whole content of the Vessell : All this performed, *when the vessell is above halfe full,* the content of that middle Segment and halfe that remainder being added together, is the content you looke for. But, *when the vessell is not halfe full,* halfe that remainder is the content desired.

Cap. XI.

The use of the Rule of Proportion in Military Orders.

Probl. 1. *Any number of Souldiers being propounded, to order them into a square battaile of men.*

FInde (by the 12 *Probleme* aforegoing) the square-root of the number given: For, looke how much that root shall happen to be, so many Souldiers ought you to place in Ranke, and so many likewise in File, to make a square Battaile of men.

Example, Let it be required to order 573 Souldiers into a square Battaile of men: the Square-root of that number is 23.94: and therefore you are to place 23 in Ranke, and as many also in File: For Fractions are not considerable in questions that concerne *Military* Orders.

Probl. 2. *Any number of Souldiers being propounded, to order them into a double Battaile of men: viz. which may have twice so many in Ranke as in File.*

Finde out the Square-root of halfe the number given: for that root is the number of Souldiers to be placed in File: and so many more ought to be placed

placed in Ranke, to make up a double Battaile of Men.

Example, 1342 Souldiers being propounded to be put into that order: I finde 26, &c. to be the Square-root of 671 (halfe the number propounded) and thereupon conclude that 26 ought to be placed in File, and 52 in Ranke, to order so many Souldiers into a double Battaile of men.

Probl. 3. *Any number of Souldiers being given, to order them into a quadruple Battaile:* viz. *such as may have foure times so many in Ranke as in File.*

Here the Square-root of the fourth part of the number given will shew the number to be placed in File, and foure times so many are to be placed in Ranke.

So 2048 Souldiers being offered to be put into that order, 22 are to be placed in File, and 88 in Ranke. For, the fourth part of 2048 is 512, whose Square-root is 22, &c.

Probl. 4. *Any number of Souldiers being given, together with their distances in Ranke and File, to order them into a Square Battaile of ground.*

Extend the Compasses upon the Meane Line of Numbers, from the distance in File to the distance in Ranke: this done, and that extent applyed the same way, and upon the same Line from the number of Souldiers propounded, will cause the movable point to fall upon a fourth number, whose Square-root appearing at the same point upon the Great Line of Numbers) is the number of men to be placed in File: by which if you divide the whole number of Souldiers, the Quotient will shew the number of men to be placed in Ranke.

Example, 2500 men are propounded to be ordered into a square Battaile of ground, in such sort that their distance in File being seven foot, and their distance in ranke three foot, the ground whereupon they stand may be a just square: To resolve this

question,

question, extend the Compasses upon the Meane Line of Numbers downwards from 7 to 3: then (because the fourth number to be found in all likelihood will consist of foure figures) if you apply that extent the same way from 2500 in the first part of the same Line, the movable point will fall upon the fourth number you looke for, where also you may observe 32, &c. upon the second part of the great Line of Numbers, which are the number of men to be placed in File; againe, if letting that point of the Compasses remaine fixed there, you close them till the other point may reach crossewise to 1 at the beginning of the first part of the said Great Line of Numbers, that extent being applyed the same way (*viz.* downwards and acrosse) from 2500 upon the same great Line, the movable point will fall neere 76, &c. which are the number of souldiers to be placed in Ranke.

Probl. 5. *Any number of Souldiers being propounded, to order them in Ranke and File according to the reason of any two numbers given.*

This Probleme is resolved much after the same manner that the last was: For,

As the proportionall number given for the File is to that given for the Ranke: so is the number of Souldiers to a fourth number, whose roote is the number of men to be placed in Ranke, by which if you divide the whole, the quotient is the number to be placed in File.

So if 2500 souldiers were to be martialled in such order, that the number of men to be placed in File might beare such proportion to the number of men to be placed in Ranke, as 5 beares to 12: I say then, as 5 is to 12, so is 2500 to another number, whose root is 77, &c. *viz.* the number of men to be placed in Ranke, by which if the same 2500 be divided, the quotient will be 32, &c. the number of men to be placed in File.

CAP. XII.

The use of the Rule of Proportion in questions that concerne Interest and Annuities.

Probl. 1. *A summe of Money being forborne for a certaine time, to finde how much it will be augmented at the expiration of the same time, accounting Interest upon Interest according to a certaine rate propounded.*

THe extent upon the Line of Numbers from 100 *l.* to the aggregate of 100 *l.* and the rate added together, being repeated the same way from the summe given, so many times as there are yeares in the question, will at last cause the movable point to fall upon the Principall increased with the Interest, according to the forbearance and rate propounded.

Example, I desire to know how much 273 *l.* being forborne for 5 yeares will be increased at the expiration of those yeares according to Interest upon

on Intereſt, and the rate of 8 *l. per centum*: Extend the Compaſſes upon the Great Line of Numbers from 100 to 108: This done, if that extent be repeated five times from 273, the movable point will at laſt fall upon 402. 1 (*viz.* 402 *l.* 2 *s.*) the Principall augmented with the Intereſt for the forbearance of thoſe five yeares.

Probl. 2. *A ſumme of Money being due at a time to come, to finde what it is worth in ready money.*

This is the *Inverſe* of the laſt: For here, if you apply that extent backwards from the number propounded, ſo many times as there are yeares in the queſtion, you ſhall have your deſire.

Example, 402 *l.* 2 *s.* being due at the end of five yeares yet to come, I deſire to know how much that ſumme is worth in ready money according to the rate of 8 *l. per centum*: Extend the Compaſſes from 100 to 108, as before: And then, if you apply that extent five times downwards from 402. 1, the movable point will at laſt fall upon 273 *l.* the value of 402. 1, in ready money.

Probl. 3. *A yearly Rent or Annuity being forborne a certaine number of yeares, to finde what the arearages thereof will amount unto according to any rate propounded.*

Firſt diſcover the Principall that anſwers to the Rent or Annuity in queſtion, then finde unto what ſumme that Principall will be augmented (according to the given rate) at the end of the terme propounded: This done, if you ſubtract the ſame Principall out of that ſumme, the remainder is the ſum of the arearages you looke for.

Example, a Rent or Annuity of 12 *l. per annum* being forborne 16 yeares, what will the arearages thereof amount unto, they being conceived to increaſe (as they grow due) after the rate of 8 *l. per centum*? Here firſt, to finde the Principall that anſwers to 12 *l.* ſay thus: If 8 *l.* hath 100 *l.* for his Princi-

Principall, what ought 12 *l.* to have for his? the answer will be (by the 4 *Probleme* of the 4 Chapter) 150 *l.* Having thus discovered the Principall of 12 *l. viz.* 150. *l.* I finde (by the first *Probleme* of this Chapter) that the same 150 *l.* being forborne 16 yeares will amount (after the rate of 8 *l. per centum*) to 513. 9, that is 513 *l.* 18 *s.* Now therefore if I deduct 150 *l.* (the correspondent Principall to the Annuity given) out of 513 *l.* 18 *s.* the remainder, *viz.* 363 *l.* 18 *s.* is the summe of the arearages required.

Probl. 4. *A yearly Rent or Annuity being propounded, to finde what it is worth in ready money.*

First finde what the arearages thereof amount unto at the end of the terme propounded, and then what those arearages are worth in ready money, which shall likewise be the required price or value of the Rent or Annuity propounded.

Example, what may a man which is desirous to lay out his money after the rate of 8 *l. per centum* afford to give for a Lease of 12 *l. per annum,* that hath yet 16 yeares in being? I finde (by the last *Probleme*) that the arearages of 12 *l. per annum,* being forborne 16 yeares, amount then unto 363 *l.* 18 *s.* or 363. 9, and I finde likewise (by the second *Probleme* aforegoing) that the same 363 *l.* 18 *s.* is worth in present money 106. 2, or (which is all one) 106 *l.* 4 *s.* I conclude therefore that the value of the Lease propounded (at the rate of 8 *l. per centum*) is 106 *l.* 4 *s.*

Here, when the terme of the Annuity begins not presently, but after certaine yeares to come, finde what the arearages forborne for all that time are worth in ready money.

So in the *example* last premised, if the annuity of 16 yeares were not to begin till after the expiration of 5 yeares, in this case you are to enquire what the arearages (*viz.* 363 *l.* 18 *s.*) being forborne 21 years, are

are worth in ready money, which you shall likewise finde (by the second *Probleme* before cited) to be 72.3, which being reduced is 72 *l.* 6 *s.* the value of the Lease required.

Probl. 5. *A sum of Money being propounded, to finde what Annuity (to continue any number of yeares, and according to any rate given) that sum will buy.*

Take any annuity at pleasure, then finde the value of that annuity in ready money: This done, the *proportion* will be as followeth:

As the value found is to the annuity taken; so is the sum given to the annuity required.

Example, What annuity (to continue 16 yeares) will 1205 *l.* deserve, so that the purchaser may gaine after the rate of 8 *l.* *per centum*? Here first I take 12 *l.* *per annum* to continue 16 yeares, and finde the value thereof in ready money (by the last *Probleme*) to be 106.2, or 106 *l.* 4 *s.* I say therefore,

If 106.2 give 12 *l.* *per annum*,

What will 1205 *l.* yeeld? *Facit* 171.4 *per annum*, which being reduced is 171 *l.* 8 *s.* I conclude therefore, that 171 *l.* 8 *s.* is the annuity (to endure 16 yeares) which 1205 *l.* doth deserve, after the rate of 8 *l.* *per centum*.

Deo Laus.

FINIS.

Note, that this Instrument is perfectly made in Brasse by Elias Allain, *at the signe of the Black-moore without Temple-Barre, London: And in Wood by* John Tompson *and* Anthony Tompson *in Hosier Lane.*

A Catalogue of ſuch Authors as are to be peruſed for the ready attaining of the practicall part of the Mathematiques, ſet downe in order of Place, as they are to be read.

1. THis *Authours* Arithmetique in Engliſh.

2. *Ramus* his Geometry, in *Latine.*

3. *Petiſcus* his Doctrine of Triangles, in Latine or Engliſh.

4. *Hues* his uſes of the Globes, in *Latine* or *Engliſh.*

5. *Petiſcus* his *Architectura Militaris* or Fortification, Geodæſie or Surveying, Aſtronomy, Geography, Gnomonology or Dialling, all in *Latine.*

6. Mr. *Wright* his Errors in Navigation, *Engliſh.*

7. Mr. *Gunters* uſe of the Sector and Croſſe-ſtaffe, *Engliſh.*

8. Mr. *Brigs* his Arithmetica Logarithmica, *Latine.*

9. The ſame Authors Trigonometria Britannica, *Latine.*

10. This

10. This Authors Vse of the Logarithmeticall Tables, *English*.

11. Mr. *Brigs* his Table of Logarithmes contracted by Mr. *Roe*, and the Tables of Sines and Tangents (thereunto annexed) together with the Vses of the said Tables, in Geometrie, Astronomie, and Navigation, published by this Author in *English*.

12. Mr. *Norwoods* Trigonometrie, and his application thereof in the three kindes of sailing, *English*.

13. Mr. *Fosters* Art of Dialling, *English*.

14. Mr. *Norwoods* Fortification, *English*.

For such as understand not the *Latine* tongue, the abovesaid Authors in *English* may suffice.

The abovementioned Bookes of M. Brigs, *M.* Hues *of the Globes in English, translated by M*r. Chilmead; *and those of* Edmund Wingate, *are to bee sold by* Philemon Stephens *at the gilded Lion in* Pauls *Churchyard*.

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